

# Pari-GP reference card

(PARI-GP version 2.15.5)

Note: optional arguments are surrounded by braces {}.

To start the calculator, type its name in the terminal: **gp**

To exit **gp**, type **quit**, **\q**, or **<C-D>** at prompt.

## Help

describe function	?function
extended description	??keyword
list of relevant help topics	???pattern
name of GP-1.39 function $f$ in GP-2.*	whatnow( $f$ )

## Input/Output

previous result, the result before	%, %`, %`` , etc.
$n$ -th result since startup	% $n$
separate multiple statements on line	;
extend statement on additional lines	\
extend statements on several lines	{ $seq_1$ ; $seq_2$ ;}
comment	/* ... */
one-line comment, rest of line ignored	\\ ...

## Metacommands & Defaults

set default $d$ to $val$	default({ $d$ },{ $val$ })
toggle timer on/off	#
print time for last result	##
print defaults	\d
set debug level to $n$	\g $n$
set memory debug level to $n$	\gm $n$
set $n$ significant digits / bits	\p $n$ , \pb $n$
set $n$ terms in series	\ps $n$
quit GP	\q
print the list of PARI types	\t
print the list of user-defined functions	\u
read file into GP	\r $filename$
set debuglevel for domain $D$ to $n$	setdebug( $D,n$ )

## Debugger / break loop

get out of break loop	break or <C-D>
go up/down $n$ frames	dbg_up({ $n$ }), dbg_down
set break point	breakpoint()
examine object $o$	dbg_x( $o$ )
current error data	dbg_err()
number of objects on heap and their size	getheap()
total size of objects on PARI stack	getstack()

## PARI Types & Input Formats

<b>t_INT</b> . Integers; hex, binary	$\pm 31$ ; $\pm 0x1F$ , $\pm 0b101$
<b>t_REAL</b> . Reals	$\pm 3.14$ , 6.022 E23
<b>t_INTMOD</b> . Integers modulo $m$	Mod( $n,m$ )
<b>t_FRAC</b> . Rational Numbers	$n/m$
<b>t_FFELT</b> . Elt in finite field $\mathbf{F}_q$	ffgen( $q$ , 't)
<b>t_COMPLEX</b> . Complex Numbers	$x + y * I$
<b>t_PADIC</b> . $p$ -adic Numbers	$x + O(p^k)$
<b>t_QUAD</b> . Quadratic Numbers	$x + y * \text{quadgen}(D, \{ 'w \})$
<b>t_POLMOD</b> . Polynomials modulo $g$	Mod( $f,g$ )
<b>t_POL</b> . Polynomials	$a * x^n + \dots + b$
<b>t_SER</b> . Power Series	$f + O(x^k)$
<b>t_RFRAC</b> . Rational Functions	$f/g$
<b>t_QFB</b> . Binary quadratic form	Qfb( $a,b,c$ )
<b>t_VEC/t_COL</b> . Row/Column Vectors	[ $x,y,z$ ], [ $x,y,z$ ]~
<b>t_VEC</b> integer range	[1..10]

<b>t_VECSMALL</b> . Vector of small ints	Vecsmall([ $x,y,z$ ])
<b>t_MAT</b> . Matrices	[ $a,b;c,d$ ]
<b>t_LIST</b> . Lists	List([ $x,y,z$ ])
<b>t_STR</b> . Strings	"abc"
<b>t_INFINITY</b> . $\pm\infty$	+oo, -oo

## Reserved Variable Names

$\pi \approx 3.14$ , $\gamma \approx 0.57$ , $C \approx 0.91$ , $I = \sqrt{-1}$	Pi, Euler, Catalan, I
Landau's big-oh notation	O

## Information about an Object, Precision

PARI type of object $x$	type( $x$ )
length of $x$ / size of $x$ in memory	# $x$ , sizebyte( $x$ )
real precision / bit precision of $x$	precision( $x$ ), bitprecision( $x$ )
$p$ -adic, series prec. of $x$	padicprec( $x,p$ ), serprec( $x,v$ )
current dynamic precision	getlocalprec, getlocalbitprec

## Operators

basic operations	+, - , *, / , ^ , sqr
$i \leftarrow i+1$ , $i \leftarrow i-1$ , $i \leftarrow i*j$ , ...	i++, i--, i*=j,...
Euclidean quotient, remainder	$x \backslash y$ , $x \% y$ , divrem( $x,y$ )
shift $x$ left or right $n$ bits	$x << n$ , $x >> n$ or shift( $x, \pm n$ )
multiply by $2^n$	shiftmul( $x,n$ )
comparison operators	<=, <, >=, >, ==, !=, ==, lex, cmp
boolean operators (or, and, not)	, &&, !
bit operations	bitand, bitneg, bitor, bitxor, bitnegimply
maximum/minimum of $x$ and $y$	max( $x,y$ ), min( $x,y$ )
sign of $x$ (gives $-1, 0, 1$ )	sign( $x$ )
binary exponent of $x$	exponent( $x$ )
derivative of $f$ , 2nd derivative, etc.	$f'$ , $f''$ , ...
differential operator	diffop( $f,v,d,\{n=1\}$ )
quote operator (formal variable)	'x
assignment	x = value
simultaneous assignment $x \leftarrow v[1]$ , $y \leftarrow v[2]$	[x,y] = v

## Select Components

<i>Caveat</i> : components start at index $n = 1$ .	
$n$ -th component of $x$	component( $x,n$ )
$n$ -th component of vector/list $x$	$x[n]$
components $a, a+1, \dots, b$ of vector $x$	$x[a..b]$
$(m,n)$ -th component of matrix $x$	$x[m,n]$
row $m$ or column $n$ of matrix $x$	$x[m,]$ , $x[,n]$
numerator/denominator of $x$	numerator( $x$ ), denominator( $x$ )

## Random Numbers

random integer/prime in $[0,N[$	random( $N$ ), randomprime( $N$ )
get/set random seed	getrand, setrand( $s$ )

## Conversions

to vector, matrix, vec. of small ints	Col/Vec, Mat, Vecsmall
to list, set, map, string	List, Set, Map, Str
create $(x \bmod y)$	Mod( $x,y$ )
make $x$ a polynomial of $v$	Pol( $x,\{v\}$ )
variants of Pol <i>et al.</i> , in reverse order	Polrev, Vecrev, Colrev
make $x$ a power series of $v$	Ser( $x,\{v\}$ )
convert $x$ to simplest possible type	simplify( $x$ )
object $x$ with real precision $n$	precision( $x,n$ )
object $x$ with bit precision $n$	bitprecision( $x,n$ )
set precision to $p$ digits in dynamic scope	localprec( $p$ )
set precision to $p$ bits in dynamic scope	localbitprec( $p$ )

## Character strings

convert to TeX representation	strtex( $x$ )
string from bytes / from format+args	strchr, sprintf
split string / join strings	strsplit, strjoin
convert time $t$ ms. to h, m, s, ms format	strtime( $t$ )
<b>Conjugates and Lifts</b>	
conjugate of a number $x$	conj( $x$ )
norm of $x$ , product with conjugate	norm( $x$ )
$L^p$ norm of $x$ ( $L^\infty$ if no $p$ )	normlp( $x,\{p\}$ )
square of $L^2$ norm of $x$	norml2( $x$ )
lift of $x$ from Mods and $p$ -adics	lift, centerlift( $x$ )
recursive lift	liftall
lift all <b>t_INT</b> and <b>t_PADIC</b> ( $\rightarrow$ <b>t_INT</b> )	liftint
lift all <b>t_POLMOD</b> ( $\rightarrow$ <b>t_POL</b> )	lifttpol

## Lists, Sets & Maps

<b>Sets</b> (= row vector with strictly increasing entries w.r.t. cmp)	
intersection of sets $x$ and $y$	setintersect( $x,y$ )
set of elements in $x$ not belonging to $y$	setminus( $x,y$ )
symmetric difference $x \Delta y$	setdelta( $x,y$ )
union of sets $x$ and $y$	setunion( $x,y$ )
does $y$ belong to the set $x$	setsearch( $x,y,\{flag\}$ )
set of all $f(x,y)$ , $x \in X$ , $y \in Y$	setbinop( $f,X,Y$ )
is $x$ a set ?	setisset( $x$ )

<b>Lists</b> . create empty list: $L = \text{List}()$	
append $x$ to list $L$	listput( $L,x,\{i\}$ )
remove $i$ -th component from list $L$	listpop( $L,\{i\}$ )
insert $x$ in list $L$ at position $i$	listinsert( $L,x,i$ )
sort the list $L$ in place	listsort( $L,\{flag\}$ )

<b>Maps</b> . create empty dictionary: $M = \text{Map}()$	
attach value $v$ to key $k$	mapput( $M,k,v$ )
recover value attach to key $k$ or error	mapget( $M,k$ )
is key $k$ in the dict? (set $v$ to $M(k)$ )	mapisdefined( $M,k,\{\&v\}$ )
remove $k$ from map domain	mapdelete( $M,k$ )

## GP Programming

### User functions and closures

$x,y$  are formal parameters;  $y$  defaults to Pi if parameter omitted;  $z,t$  are local variables (lexical scope),  $z$  initialized to 1.

**fun**(x, y=Pi) = my(z=1, t); seq

**fun** = (x, y=Pi) -> my(z=1, t); seq

attach help message $h$ to $s$	addhelp( $s,h$ )
undefine symbol $s$ (also kills help)	kill( $s$ )
<b>Control Statements</b> ( $X$ : formal parameter in expression $seq$ )	
if $a \neq 0$ , evaluate $seq_1$ , else $seq_2$	if( $a,\{seq_1\},\{seq_2\}$ )
eval. $seq$ for $a \leq X \leq b$	for( $X = a,b,seq$ )
...for $X \in v$	foreach( $v,X,seq$ )
...for primes $a \leq X \leq b$	forprime( $X = a,b,seq$ )
...for primes $\equiv a \pmod q$	forprimestep( $X = a,b,q,seq$ )
...for composites $a \leq X \leq b$	forcomposite( $X = a,b,seq$ )
...for $a \leq X \leq b$ stepping $s$	forstep( $X = a,b,s,seq$ )
...for $X$ dividing $n$	fordiv( $n,X,seq$ )
... $X = [n, factor(n)]$ , $a \leq n \leq b$	forfactored( $X = a,b,seq$ )
...as above, $n$ squarefree	forsquarefree( $X = a,b,seq$ )
... $X = [d, factor(d)]$ , $d   n$	fordivfactored( $n,X,seq$ )
multivariable for, lex ordering	forvec( $X = v,seq$ )

```

loop over partitions of  $n$ 
... permutations of  $S$ 
... subsets of  $\{1, \dots, n\}$ 
...  $k$ -subsets of  $\{1, \dots, n\}$ 
... vectors  $v$ ,  $q(v) \leq B$ ;  $q > 0$ 
...  $H < G$  finite abelian group
evaluate  $seq$  until  $a \neq 0$ 
while  $a \neq 0$ , evaluate  $seq$ 
exit  $n$  innermost enclosing loops
start new iteration of  $n$ -th enclosing loop
return  $x$  from current subroutine
Exceptions, warnings
raise an exception / warning
type of error message  $E$ 
try  $seq_1$ , evaluate  $seq_2$  on error
Functions with closure arguments / results
number of arguments of  $f$ 
select from  $v$  according to  $f$ 
apply  $f$  to all entries in  $v$ 
evaluate  $f(a_1, \dots, a_n)$ 
evaluate  $f(\dots f(f(a_1, a_2), a_3) \dots, a_n)$ 
calling function as closure
Sums & Products
sum  $X = a$  to  $X = b$ , initialized at  $x$ 
sum entries of vector  $v$ 
product of all vector entries
sum  $expr$  over divisors of  $n$ 
... assuming  $expr$  multiplicative
product  $a \leq X \leq b$ , initialized at  $x$ 
product over primes  $a \leq X \leq b$ 
Sorting
sort  $x$  by  $k$ -th component
min.  $m$  of  $x$  ( $m = x[i]$ ), max.
does  $y$  belong to  $x$ , sorted wrt.  $f$ 
 $\prod g^x \rightarrow$  factorization ( $\Rightarrow$  sorted, unique  $g$ )
Input/Output
print with/without  $\backslash n$ ,  $\text{\TeX}$  format
pretty print matrix
print fields with separator
formatted printing
write  $args$  to file
write  $x$  in binary format
read file into GP
... return as vector of lines
... return as vector of strings
read a string from keyboard
Files and file descriptors
File descriptors allow efficient small consecutive reads or writes
from or to a given file. The argument  $n$  below is always a descriptor,
attached to a file in r(ead), w(rite) or a(ppend) mode.
get descriptor  $n$  for file  $path$  in given  $mode$ 
... from shell  $cmd$  output (pipe)
close descriptor
commit pending write operations
read logical line from file
... raw line from file
write  $s \backslash n$  to file
... write  $s$  to file

```

```

forpart( $p = n, seq$ )
forperm( $S, p, seq$ )
forsubset( $n, p, seq$ )
forsubset( $[n, k], p, seq$ )
forqfvec( $v, q, b, seq$ )
forsubgroup( $H = G$ )
until( $a, seq$ )
while( $a, seq$ )
break( $\{n\}$ )
next( $\{n\}$ )
return( $\{x\}$ )

error(), warning()
errname( $E$ )
iferr( $seq_1, E, seq_2$ )

Results
arity( $f$ )
select( $f, v$ )
apply( $f, v$ )
call( $f, a$ )
fold( $f, a$ )
self()

sum( $X = a, b, expr, \{x\}$ )
vecsum( $v$ )
vecprod( $v$ )
sumdiv( $n, X, expr$ )
sumdivmult( $n, X, expr$ )
prod( $X = a, b, expr, \{x\}$ )
prodeuler( $X = a, b, expr$ )

vecsort( $x, \{k\}, \{fl = 0\}$ )
vecmin( $x, \{\&i\}$ ), vecmax
vecsearch( $x, y, \{f\}$ )
matreduce( $m$ )

print, print1, printtex
printp
printsep( $sep, \dots$ ), printsep1
printf()

write, write1, writetex( $file, args$ )
writebin( $file, x$ )
read( $\{file\}$ )
readvec( $\{file\}$ )
readstr( $\{file\}$ )
input()

fileclose( $n$ )
fileflush( $n$ )
fileread( $n$ )
filereadstr( $n$ )
filewrite( $n, s$ )
filewrite1( $n, s$ )

```

# Pari-GP reference card

(PARI-GP version 2.15.5)

## Timers

CPU time in  $ms$  and reset timer  
CPU time in  $ms$  since gp startup  
time in  $ms$  since UNIX Epoch  
timeout command after  $s$  seconds

## Interface with system

allocates a new stack of  $s$  bytes  
alias  $old$  to  $new$   
install function from library  
execute system command  $a$   
... and feed result to GP  
... returning GP string  
get  $\$VAR$  from environment  
expand env. variable in string

## Parallel evaluation

These functions evaluate their arguments in parallel (pthreads or MPI); args. must not access global variables (use **export** for this) and must be free of side effects. Enabled if threading engine is not *single* in gp header.

evaluate  $f$  on  $x[1], \dots, x[n]$   
evaluate closures  $f[1], \dots, f[n]$   
as **select**  
as **sum**  
as **vector**  
eval  $f$  for  $i = a, \dots, b$   
... for each element  $x$  in  $v$   
... for  $p$  prime in  $[a, b]$   
... for  $p = a \bmod q$   
... multivariate  
export  $x$  to parallel world  
... all dynamic variables  
frees exported value  $x$   
... all exported values

## Linear Algebra

dimensions of matrix  $x$   
multiply two matrices  
... assuming result is diagonal  
concatenation of  $x$  and  $y$   
extract components of  $x$   
transpose of vector or matrix  $x$   
adjoint of the matrix  $x$   
eigenvectors/values of matrix  $x$   
characteristic/minimal polynomial of  $x$   
trace/determinant of matrix  $x$   
permanent of matrix  $x$   
Frobenius form of  $x$   
QR decomposition  
apply **matqr**'s transform to  $v$

## Constructors & Special Matrices

$\{g(x): x \in v \text{ s.t. } f(x)\}$   
 $\{x: x \in v \text{ s.t. } f(x)\}$   
 $\{g(x): x \in v\}$   
row vec. of  $expr$  eval'ed at  $1 \leq i \leq n$   
col. vec. of  $expr$  eval'ed at  $1 \leq i \leq n$   
vector of small ints

gettime()  
getabstime()  
getwalltime()  
alarm( $s, expr$ )  
allocatemem( $\{s\}$ )  
alias( $new, old$ )  
install( $f, code, \{gpf\}, \{lib\}$ )  
system( $a$ )  
extern( $a$ )  
externstr( $a$ )  
getenv("VAR")  
strexpend( $x$ )

parapply( $f, x$ )  
pareval( $f$ )  
parselect( $f, A, \{flag\}$ )  
parsum( $i = a, b, expr$ )  
parvector( $n, i, \{expr\}$ )  
parfor( $i = a, \{b\}, f, \{r\}, \{f_2\}$ )  
parforeach( $v, x, f, \{r\}, \{f_2\}$ )  
parforprime( $p = a, \{b\}, f, \{r\}, \{f_2\}$ )  
parforprimestep( $p = a, \{b\}, q, f, \{r\}, \{f_2\}$ )  
parforvec( $X = v, f, \{r\}, \{f_2\}, \{flag\}$ )  
export( $x$ )  
exportall()  
unexport( $x$ )  
unexportall()

matsize( $x$ )  
 $x * y$   
matmultodiagonal( $x, y$ )  
concat( $x, \{y\}$ )  
vecextract( $x, y, \{z\}$ )  
 $x \sim$ , mattranspose( $x$ )  
matadjoint( $x$ )  
mateigen( $x$ )  
charpoly( $x$ ), minpoly( $x$ )  
trace( $x$ ), matdet( $x$ )  
matpermanent( $x$ )  
matfrobenius( $x$ )  
matqr( $x$ )  
mathouseholder( $Q, v$ )

$[g(x) \mid x \leftarrow v, f(x)]$   
 $[x \mid x \leftarrow v, f(x)]$   
 $[g(x) \mid x \leftarrow v]$   
vector( $n, \{i\}, \{expr\}$ )  
vectorv( $n, \{i\}, \{expr\}$ )  
vectorsmall( $n, \{i\}, \{expr\}$ )

$[c, c \cdot x, \dots, c \cdot x^n]$   
 $[1, 2^x, \dots, n^x]$   
matrix  $1 \leq i \leq m, 1 \leq j \leq n$   
define matrix by blocks  
diagonal matrix with diagonal  $x$   
is  $x$  diagonal?  
 $x \cdot \text{matdiagonal}(d)$   
 $n \times n$  identity matrix  
Hessenberg form of square matrix  $x$   
 $n \times n$  Hilbert matrix  $H_{ij} = (i + j - 1)^{-1}$   
 $n \times n$  Pascal triangle  
companion matrix to polynomial  $x$   
Sylvester matrix of  $x$  and  $y$

## Gaussian elimination

kernel of matrix  $x$   
intersection of column spaces of  $x$  and  $y$   
solve  $MX = B$  ( $M$  invertible)  
one sol of  $M * X = B$   
basis for image of matrix  $x$   
columns of  $x$  *not* in **matimage**  
supplement columns of  $x$  to get basis  
rows, cols to extract invertible matrix  
rank of the matrix  $x$   
solve  $MX = B \bmod D$   
image mod  $D$   
kernel mod  $D$   
inverse mod  $D$   
determinant mod  $D$

## Lattices & Quadratic Forms

### Quadratic forms

evaluate  ${}^t x Q y$   
evaluate  ${}^t x Q x$   
signature of quad form  ${}^t y * x * y$   
decomp into squares of  ${}^t y * x * y$   
eigenvalues/vectors for real symmetric  $x$

### HNF and SNF

upper triangular Hermite Normal Form  
HNF of  $x$  where  $d$  is a multiple of  $\det(x)$   
multiple of  $\det(x)$   
HNF of  $(x \mid \text{diagonal}(D))$   
elementary divisors of  $x$   
 $q$ -rank from elementary divisors  
elementary divisors of  $\mathbf{Z}[a]/(f'(a))$   
integer kernel of  $x$   
 $\mathbf{Z}$ -module  $\leftrightarrow$   $\mathbf{Q}$ -vector space

### Lattices

LLL-algorithm applied to columns of  $x$   
... for Gram matrix of lattice  
find up to  $m$  sols of **qfnorm**( $x, y) \leq b$   
 $v, v[i] :=$  number of  $y$  s.t. **qfnorm**( $x, y) = i$   
perfection rank of  $x$   
find isomorphism between  $q$  and  $Q$   
precompute for isomorphism test with  $q$   
automorphism group of  $q$

Based on an earlier version by Joseph H. Silverman  
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Send comments and corrections to (Karim.Belabas@math.u-bordeaux.fr)

convert `qfauto` for GAP/Magma `qfautoexport(G, {flag})`  
orbits of  $V$  under  $G \subset \mathrm{GL}(V)$  `qforbits(G, V)`

Polynomials & Rational Functions

all defined polynomial variables `variables()`  
get var. of highest priority (higher than  $v$ ) `varhigher(name, {v})`  
... of lowest priority (lower than  $v$ ) `varlower(name, {v})`

Coefficients, variables and basic operators

degree of  $f$  `poldegree(f)`  
coef. of degree  $n$  of  $f$ , leading coef. `polcoef(f, n)`, `pollead`  
main variable / all variables in  $f$  `variable(f)`, `variables(f)`  
replace  $x$  by  $y$  in  $f$  `subst(f, x, y)`  
evaluate  $f$  replacing vars by their value `eval(f)`  
replace polynomial expr.  $T(x)$  by  $y$  in  $f$  `substpol(f, T, y)`  
replace  $x_1, \dots, x_n$  by  $y_1, \dots, y_n$  in  $f$  `substvec(f, x, y)`

$f \in A[x]$ ; reciprocal polynomial  $x^{\deg f} f\left(\frac{1}{x}\right)$  `polrecip(f)`  
gcd of coefficients of  $f$  `content(f)`  
derivative of  $f$  w.r.t.  $x$  `deriv(f, {x})`  
...  $n$ -th derivative of  $f$  `derivn(f, n, {x})`  
formal integral of  $f$  w.r.t.  $x$  `intformal(f, {x})`  
formal sum of  $f$  w.r.t.  $x$  `sumformal(f, {x})`

Constructors & Special Polynomials

interpolation polynomial at  $(x[1], y[1]), \dots, (x[n], y[n])$ , evaluated at  $t$ , with error estimate  $e$  `polinterpolate(x, {y}, {t}, {&e})`  
 $T_n/U_n, H_n$  `polchebyshev(n)`, `polhermite(n)`  
 $P_n, L_n^{(\alpha)}$  `pollegendre(n)`, `pollaguerre(n, a)`  
 $n$ -th cyclotomic polynomial  $\Phi_n$  `polcyclo(n)`  
return  $n$  if  $f = \Phi_n$ , else 0 `poliscyclo(f)`  
is  $f$  a product of cyclotomic polynomials? `poliscycloprod(f)`  
Zagier's polynomial of index  $(n, m)$  `polzagier(n, m)`

Resultant, elimination

discriminant of polynomial  $f$  `poldisc(f)`  
find factors of `poldisc(f)` `poldiscfactors(f)`  
resultant  $R = \mathrm{Res}_v(f, g)$  `polresultant(f, g, {v})`  
 $[u, v, R], xu + yv = \mathrm{Res}_v(f, g)$  `polresultanttext(x, y, {v})`  
solve Thue equation  $f(x, y) = a$  `thue(t, a, {sol})`  
initialize  $t$  for Thue equation solver `thueinit(f)`

Roots and Factorization (Complex/Real)

complex roots of  $f$  `polroots(f)`  
bound complex roots of  $f$  `polrootsbound(f)`  
number of real roots of  $f$  (in  $[a, b]$ ) `polsturm(f, {[a, b]})`  
real roots of  $f$  (in  $[a, b]$ ) `polrootsreal(f, {[a, b]})`  
complex embeddings of  $\mathbf{t\_POLMOD} \ z$  `conjsvec(z)`

Roots and Factorization (Finite fields)

factor  $f$  mod  $p$ , roots `factormod(f, p)`, `polrootsmod`  
factor  $f$  over  $\mathbf{F}_p[x]/(T)$ , roots `factormod(f, [T, p])`, `polrootsmod`  
squarefree factorization of  $f$  in  $\mathbf{F}_q[x]$  `factormodSQF(f, {D})`  
distinct degree factorization of  $f$  in  $\mathbf{F}_q[x]$  `factormodDDF(f, {D})`  
factor  $n$ -th cyclotomic pol.  $\Phi_n$  mod  $p$  `factormodcyclo(n, p)`

Roots and Factorization ( $p$ -adic fields)

factor  $f$  over  $\mathbf{Q}_p$ , roots `factorpadic(f, p, r)`, `polrootspadic`  
 $p$ -adic root of  $f$  congruent to  $a$  mod  $p$  `padicappr(f, a)`  
Newton polygon of  $f$  for prime  $p$  `newtonpoly(f, p)`  
Hensel lift  $A/\mathrm{lc}(A) = \prod_i B[i] \bmod p^e$  `polhensellift(A, B, p, e)`  
 $T = \prod (x - z_i) \mapsto \prod [x - \omega(z_i)] \in \mathbf{Z}_p[x]$  `polteichmuller(T, p, e)`  
extensions of  $\mathbf{Q}_p$  of degree  $N$  `padicfields(p, N)`

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Roots and Factorization (Miscellaneous)

symmetric powers of roots of  $f$  up to  $n$  `polsym(f, n)`  
Graeffe transform of  $f$ ,  $g(x^2) = f(x)f(-x)$  `polgraeffe(f)`  
factor  $f$  over coefficient field `factor(f)`  
cyclotomic factors of  $f \in \mathbf{Q}[X]$  `polcyclofactors(f)`

Finite Fields

A finite field is encoded by any element (`t_FFELT`).  
find irreducible  $T \in \mathbf{F}_p[x]$ ,  $\deg T = n$  `ffinit(p, n, {x})`  
Create  $t$  in  $\mathbf{F}_q \simeq \mathbf{F}_p[t]/(T)$  `t = ffgen(T, 't)`  
... indirectly, with implicit  $T$  `t = ffgen(q, 't); T = t.mod`  
map  $m$  from  $\mathbf{F}_q \ni a$  to  $\mathbf{F}_{q^k} \ni b$  `m = ffbend(a, b)`  
build  $K = \mathbf{F}_q[x]/(P)$  extending  $\mathbf{F}_q \ni a$ , `ffextend(a, P)`  
evaluate map  $m$  on  $x$  `ffmap(m, x)`  
inverse map of  $m$  `ffinvmap(m)`  
compose maps  $m \circ n$  `ffcompomap(m, n)`  
 $x$  as polmod over codomain of map  $m$  `ffmaprel(m, x)`  
 $F^n$  over  $\mathbf{F}_q \ni a$  `fffrobenius(a, n)`  
 $\#$ {monic irred.  $T \in \mathbf{F}_q[x]$ ,  $\deg T = n$ } `ffnbirred(q, n)`

Formal &  $p$ -adic Series

truncate power series or  $p$ -adic number `truncate(x)`  
valuation of  $x$  at  $p$  `valuation(x, p)`  
**Dirichlet and Power Series**  
Taylor expansion around 0 of  $f$  w.r.t.  $x$  `taylor(f, x)`  
Laurent series of closure  $F$  up to  $x^k$  `laurentseries(f, k)`  
 $\sum a_k b_k t^k$  from  $\sum a_k t^k$  and  $\sum b_k t^k$  `serconvol(a, b)`  
 $f = \sum a_k t^k$  from  $\sum (a_k/k!) t^k$  `serlaplace(f)`  
reverse power series  $F$  so  $F(f(x)) = x$  `serreverse(f)`  
remove terms of degree  $< n$  in  $f$  `serchop(f, n)`  
Dirichlet series multiplication / division `dirmul, dirdiv(x, y)`  
Dirichlet Euler product ( $b$  terms) `direuler(p = a, b, expr)`

Transcendental and  $p$ -adic Functions

real, imaginary part of  $x$  `real(x)`, `imag(x)`  
absolute value, argument of  $x$  `abs(x)`, `arg(x)`  
square/ $n$ th root of  $x$  `sqrt(x)`, `sqrtn(x, n, {&z})`  
all  $n$ -th roots of 1 `rootsof1(n)`  
FFT of  $[f_0, \dots, f_{n-1}]$  `w = fftinit(n)`, `fft/fftinw(w, f)`  
trig functions `sin, cos, tan, cotan, sinc`  
inverse trig functions `asin, acos, atan`  
hyperbolic functions `sinh, cosh, tanh, cotanh`  
inverse hyperbolic functions `asinh, acosh, atanh`  
 $\log(x)$ ,  $\log(1+x)$ ,  $e^x$ ,  $e^x - 1$  `log, loglp, exp, expm1`  
Euler  $\Gamma$  function,  $\log \Gamma$ ,  $\Gamma'/\Gamma$  `gamma, lngamma, psi`  
half-integer gamma function  $\Gamma(n+1/2)$  `gammah(n)`  
Riemann's zeta  $\zeta(s) = \sum n^{-s}$  `zeta(s)`  
 $\sum_{1 \leq n \leq N} n^s$  `dirpowerssum(N, s)`  
Hurwitz's  $\zeta(s, x) = \sum (n+x)^{-s}$  `zetahurwitz(s, x)`  
Lerch  $\Phi(z, s, x) = \sum z^n (n+x)^{-s}$  `lerchphi(z, s, x)`  
Lerch  $L(s, x, t) = \Phi(e^{2i\pi t}, s, x)$  `lerchzeta(s, x, t)`  
multiple zeta value (MZV),  $\zeta(s_1, \dots, s_k)$  `zetamult(s, {T})`  
all MZVs for weight  $\sum s_i = n$  `zetamultall(n)`  
convert MZV id to  $[s_1, \dots, s_k]$  `zetamultconvert(f, {flag})`  
MZV dual sequence `zetamultdual(s)`  
multiple polylog  $Li_{s_1, \dots, s_k}(z_1, \dots, z_k)$  `polylogmult(s, z)`

incomplete  $\Gamma$  function ( $y = \Gamma(s)$ ) `incgam(s, x, {y})`  
complementary incomplete  $\Gamma$  `incgamc(s, x)`  
 $\int_x^\infty e^{-t} dt/t$ ,  $(2/\sqrt{\pi}) \int_x^\infty e^{-t^2} dt$  `eint1, erfc`  
elliptic integral of 1st and 2nd kind `ellK(k)`, `ellE(k)`  
dilogarithm of  $x$  `dilog(x)`  
 $m$ -th polylogarithm of  $x$  `polylog(m, x, {flag})`  
 $U$ -confluent hypergeometric function `hyperu(a, b, u)`  
Hypergeometric  ${}_pF_q(A, B; z)$  `hypergeom(A, B, z)`  
Bessel  $J_n(x)$ ,  $J_{n+1/2}(x)$  `besselj(n, x)`, `besseljh(n, x)`  
Bessel  $I_\nu, K_\nu, H_\nu^1, H_\nu^2, Y_\nu$  `(bessel)i, k, h1, h2, y`  
 $k$ -th zero of  $J_\nu(x)$  `besseljzero(nu, {k = 1})`  
 $k$ -th zero of  $Y_\nu(x)$  `besselyzero(nu, {k = 1})`  
Airy functions  $A_i(x)$ ,  $B_i(x)$  `airy(x)`  
Lambert  $W$ :  $x$  s.t.  $xe^x = y$  `lambertw(y)`  
Teichmuller character of  $p$ -adic  $x$  `teichmuller(x)`

Iterations, Sums & Products

Numerical integration for meromorphic functions

Behaviour at endpoint for Double Exponential (DE) methods: either a scalar ( $a \in \mathbf{C}$ , regular) or  $\pm\infty$  (decreasing at least as  $x^{-2}$ ) or  
 $(x-a)^{-\alpha}$  singularity `[a, a]`  
exponential decrease  $e^{-\alpha|x|}$  `[ $\pm\infty$ , a],  $\alpha > 0$`   
slow decrease  $|x|^\alpha$  `...  $\alpha < -1$`   
oscillating as  $\cos(kx)$   `$\alpha = k\mathbf{I}$ ,  $k > 0$`   
oscillating as  $\sin(kx)$   `$\alpha = -k\mathbf{I}$ ,  $k > 0$`

numerical integration `intnum(x = a, b, f, {T})`  
weights  $T$  for `intnum` `intnuminit(a, b, {m})`  
weights  $T$  incl. kernel  $K$  `intfuncinit(t = a, b, K, {m})`  
integrate  $(2i\pi)^{-1} f$  on circle  $|z-a| = R$  `intcirc(x = a, R, f, {T})`  
**Other integration methods**  
 $n$ -point Gauss-Legendre `intnumgauss(x = a, b, f, {n})`  
weights for  $n$ -point Gauss-Legendre `intnumgaussinit({n})`  
quasi-periodic function, period  $2H$  `intnumosc(x = a, f, H)`  
Romberg (low accuracy) `intnumromb(x = a, b, f, {flag})`

Numerical summation

sum of series  $f(n)$ ,  $n \geq a$  (low accuracy) `suminf(n = a, expr)`  
sum of alternating/positive series `sumalt, sumpos`  
sum of series using Euler-Maclaurin `sumnum(n = a, f, {T})`  
... Sidi summation `sumnumsidi(n = a, f)`  
 $\sum_{n \geq a} F(n)$ ,  $F$  rational function `sumnumrat(F, a)`  
...  $\sum_{p \geq a} F(p^s)$  `sumeulerrat(F, {s = 1}, {a = 2})`  
weights for `sumnum`,  $a$  as in DE `sumnuminit({ $\infty$ , a})`  
sum of series by Monien summation `sumnummonien(n = a, f, {T})`  
weights for `sumnummonien` `sumnummonieninit({ $\infty$ , a})`  
sum of series using Abel-Plana `sumnumap(n = a, f, {T})`  
weights for `sumnumap`,  $a$  as in DE `sumnumapinit({ $\infty$ , a})`  
sum of series using Lagrange `sumnumlagrange(n = a, f, {T})`  
weights for `sumnumlagrange` `sumnumlagrangeinit`

Products

product  $a \leq X \leq b$ , initialized at  $x$  `prod(X = a, b, expr, {x})`  
product over primes  $a \leq X \leq b$  `prodeuler(X = a, b, expr)`  
infinite product  $a \leq X \leq \infty$  `prodinf(X = a, expr)`  
 $\prod_{n \geq a} F(n)$ ,  $F$  rational function `prodnumrat(F, a)`  
 $\prod_{p \geq a} F(p^s)$  `prodeulerrat(F, {s = 1}, {a = 2})`

Other numerical methods

real root of $f$ in $[a, b]$ ; bracketed root	<code>solve(<math>X = a, b, f</math>)</code>
...interval splitting, step $s$	<code>solvestep(<math>X = a, b, s, f, \{flag = 0\}</math>)</code>
limit of $f(t)$ , $t \rightarrow \infty$	<code>limitnum(<math>f, \{\alpha\}</math>)</code>
asymptotic expansion of $f$ (rational)	<code>asypnum(<math>f, \{\alpha\}</math>)</code>
... $N + 1$ terms as floats	<code>asypnumraw(<math>f, N, \{\alpha\}</math>)</code>
numerical derivation w.r.t $x$ : $f'(a)$	<code>derivnum(<math>x = a, f</math>)</code>
evaluate continued fraction $F$ at $t$	<code>contfraceval(<math>F, t, \{L\}</math>)</code>
power series to cont. fraction ( $L$ terms)	<code>contfracinit(<math>S, \{L\}</math>)</code>
Padé approximant (deg. denom. $\leq B$ )	<code>bestapprPade(<math>S, \{B\}</math>)</code>

Elementary Arithmetic Functions

vector of binary digits of $ x $	<code>binary(<math>x</math>)</code>
bit number $n$ of integer $x$	<code>bittest(<math>x, n</math>)</code>
Hamming weight of integer $x$	<code>hammingweight(<math>x</math>)</code>
digits of integer $x$ in base $B$	<code>digits(<math>x, \{B = 10\}</math>)</code>
sum of digits of integer $x$ in base $B$	<code>sumdigits(<math>x, \{B = 10\}</math>)</code>
integer from digits	<code>fromdigits(<math>v, \{B = 10\}</math>)</code>
ceiling/floor/fractional part	<code>ceil, floor, frac</code>
round $x$ to nearest integer	<code>round(<math>x, \{\&amp;e\}</math>)</code>
truncate $x$	<code>truncate(<math>x, \{\&amp;e\}</math>)</code>
gcd/LCM of $x$ and $y$	<code>gcd(<math>x, y</math>), lcm(<math>x, y</math>)</code>
gcd of entries of a vector/matrix	<code>content(<math>x</math>)</code>

Primes and Factorization

extra prime table	<code>addprimes()</code>
add primes in $v$ to prime table	<code>addprimes(<math>v</math>)</code>
remove primes from prime table	<code>removeprimes(<math>v</math>)</code>
Chebyshev $\pi(x)$ , $n$ -th prime $p_n$	<code>primepi(<math>x</math>), prime(<math>n</math>)</code>
vector of first $n$ primes	<code>primes(<math>n</math>)</code>
smallest prime $\geq x$	<code>nextprime(<math>x</math>)</code>
largest prime $\leq x$	<code>precprime(<math>x</math>)</code>
factorization of $x$	<code>factor(<math>x, \{lim\}</math>)</code>
...selecting specific algorithms	<code>factorint(<math>x, \{flag = 0\}</math>)</code>
$n = df^2$ , $d$ squarefree/fundamental	<code>core(<math>n, \{fl\}</math>), coredisc</code>
certificate for (prime) $N$	<code>primecert(<math>N</math>)</code>
verifies a certificate $c$	<code>primecertisvalid(<math>c</math>)</code>
convert certificate to Magma/PRIMO	<code>primecertexport</code>
recover $x$ from its factorization	<code>factorback(<math>f, \{e\}</math>)</code>
$x \in \mathbf{Z}$ , $ x  \leq X$ , $\gcd(N, P(x)) \geq N$	<code>zncoppersmith(<math>P, N, X, \{B\}</math>)</code>
divisors of $N$ in residue class $r$ mod $s$	<code>divisorslensstra(<math>N, r, s</math>)</code>

Divisors and multiplicative functions

number of prime divisors $\omega(n)$ / $\Omega(n)$	<code>omega(<math>n</math>), bigomega</code>
divisors of $n$ / number of divisors $\tau(n)$	<code>divisors(<math>n</math>), numdiv</code>
sum of ( $k$ -th powers of) divisors of $n$	<code>sigma(<math>n, \{k\}</math>)</code>
Möbius $\mu$ -function	<code>moebius(<math>x</math>)</code>
Ramanujan's $\tau$ -function	<code>ramanujantau(<math>x</math>)</code>

Combinatorics

factorial of $x$	<code>x!</code> or <code>factorial(<math>x</math>)</code>
binomial coefficient $\binom{x}{k}$	<code>binomial(<math>x, \{k\}</math>)</code>
Bernoulli number $B_n$ as real/rational	<code>bernreal(<math>n</math>), bernfrac</code>
$[B_0, B_2, \dots B_{2k}]$	<code>bernvec(<math>k</math>)</code>
Bernoulli polynomial $B_n(x)$	<code>bernpol(<math>n, \{x\}</math>)</code>
Euler numbers	<code>eulerfrac, eulerreal, eulervec</code>
Euler polynomial $E_n(x)$	<code>eulerpol(<math>n, \{x\}</math>)</code>
Eulerian polynomial $A_n(x)$	<code>eulerianpol</code>
Fibonacci number $F_n$	<code>fibonacci(<math>n</math>)</code>
Harmonic number $H_{n,r} = 1^{-r} + \dots + n^{-r}$	<code>harmonic(<math>n, r</math>)</code>
Stirling numbers $s(n, k)$ and $S(n, k)$	<code>stirling(<math>n, k, \{flag\}</math>)</code>

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number of partitions of $n$	<code>numbpart(<math>n</math>)</code>
$k$ -th permutation on $n$ letters	<code>numtoperm(<math>n, k</math>)</code>
...index $k$ of permutation $v$	<code>permtotnum(<math>v</math>)</code>
order of permutation $p$	<code>permorder(<math>p</math>)</code>
signature of permutation $p$	<code>permsign(<math>p</math>)</code>
cyclic decomposition of permutation $p$	<code>permcycles(<math>p</math>)</code>

Multiplicative groups  $(\mathbf{Z}/N\mathbf{Z})^*$ ,  $\mathbf{F}_q^*$

Euler $\phi$ -function	<code>eulerphi(<math>x</math>)</code>
multiplicative order of $x$ (divides $\phi$ )	<code>znorder(<math>x, \{o\}</math>), fford</code>
primitive root mod $q$ / $x$ .mod	<code>znprimroot(<math>q</math>), fprimroot(<math>x</math>)</code>
structure of $(\mathbf{Z}/n\mathbf{Z})^*$	<code>znstar(<math>n</math>)</code>
discrete logarithm of $x$ in base $g$	<code>znlog(<math>x, g, \{o\}</math>), fflag</code>
Kronecker-Legendre symbol $(\frac{x}{y})$	<code>kronecker(<math>x, y</math>)</code>
quadratic Hilbert symbol (at $p$ )	<code>hilbert(<math>x, y, \{p\}</math>)</code>

Euclidean algorithm, continued fractions

CRT: solve $z \equiv x$ and $z \equiv y$	<code>chinese(<math>x, y</math>)</code>
minimal $u, v$ so $xu + yv = \gcd(x, y)$	<code>gcdext(<math>x, y</math>)</code>
half-gcd algorithm	<code>halfgcd(<math>x, y</math>)</code>
continued fraction of $x$	<code>confrac(<math>x, \{b\}, \{lmax\}</math>)</code>
last convergent of continued fraction $x$	<code>confracpnqn(<math>x</math>)</code>
rational approximation to $x$ (den. $\leq B$ )	<code>bestappr(<math>x, \{B\}</math>)</code>
recognize $x \in \mathbf{C}$ as polmod mod $T \in \mathbf{Z}[X]$	<code>bestapprnf(<math>x, T</math>)</code>

Miscellaneous

integer square / $n$ -th root of $x$	<code>sqrtnint(<math>x, n</math>)</code>
largest integer $e$ s.t. $b^e \leq x$ , $e = \lfloor \log_b(x) \rfloor$	<code>logint(<math>x, b, \{\&amp;z\}</math>)</code>

Characters

Let  $\chi = [d_1, \dots, d_k]$  represent an abelian group  $G = \oplus (\mathbf{Z}/d_j\mathbf{Z}) \cdot g_j$  or any structure  $G$  affording a `.cyc` method; e.g. `znstar( $q, 1$ )` for Dirichlet characters. A character  $\chi$  is coded by  $[c_1, \dots, c_k]$  such that  $\chi(g_j) = e(n_j/d_j)$ .  
 $\chi \cdot \psi$ ;  $\chi^{-1}$ ;  $\chi \cdot \psi^{-1}$ ;  $\chi^k$       `charmul, charconj, chardiv, charpow`  
order of  $\chi$       `charorder( $cyc, \chi$ )`  
kernel of  $\chi$       `charker( $cyc, \chi$ )`  
 $\chi(x)$ ,  $G$  a GP group structure      `chareval( $G, \chi, x, \{z\}$ )`  
Galois orbits of characters      `chargalois( $G$ )`

Dirichlet Characters

initialize $G = (\mathbf{Z}/q\mathbf{Z})^*$	<code>G = znstar(<math>q, 1</math>)</code>
convert datum $D$ to $[G, \chi]$	<code>znchar(<math>D</math>)</code>
is $\chi$ odd?	<code>zncharisodd(<math>G, \chi</math>)</code>
real $\chi \rightarrow$ Kronecker symbol $(D/\cdot)$	<code>znchartokronecker(<math>G, \chi</math>)</code>
conductor of $\chi$	<code>zncharconductor(<math>G, \chi</math>)</code>
$[G_0, \chi_0]$ primitive attached to $\chi$	<code>znchartoprimitive(<math>G, \chi</math>)</code>
induce $\chi \in \hat{G}$ to $\mathbf{Z}/N\mathbf{Z}$	<code>zncharinduce(<math>G, \chi, N</math>)</code>
$\chi p$	<code>znchardecompose(<math>G, \chi, p</math>)</code>
$\prod_p  (\chi, N)  \chi p$	<code>znchardecompose(<math>G, \chi, Q</math>)</code>
complex Gauss sum $G_a(\chi)$	<code>znchargauss(<math>G, \chi</math>)</code>

Conrey labelling

Conrey label $m \in (\mathbf{Z}/q\mathbf{Z})^* \rightarrow$ character	<code>znconreychar(<math>G, m</math>)</code>
character $\rightarrow$ Conrey label	<code>znconreyexp(<math>G, \chi</math>)</code>
log on Conrey generators	<code>znconreylog(<math>G, m</math>)</code>
conductor of $\chi$ ( $\chi_0$ primitive)	<code>znconreyconductor(<math>G, \chi, \{\chi_0\}</math>)</code>

True-False Tests

is $x$ the disc. of a quadratic field?	<code>isfundamental(<math>x</math>)</code>
is $x$ a prime?	<code>isprime(<math>x</math>)</code>
is $x$ a strong pseudo-prime?	<code>ispseudoprime(<math>x</math>)</code>
is $x$ square-free?	<code>issquarefree(<math>x</math>)</code>
is $x$ a square?	<code>issquare(<math>x, \{\&amp;n\}</math>)</code>
is $x$ a perfect power?	<code>ispower(<math>x, \{k\}, \{\&amp;n\}</math>)</code>
is $x$ a perfect power of a prime? ( $x = p^n$ )	<code>isprimepower(<math>x, \&amp;n</math>)</code>
... of a pseudoprime?	<code>ispseudoprimepower(<math>x, \&amp;n</math>)</code>
is $x$ powerful?	<code>ispowerful(<math>x</math>)</code>
is $x$ a totient? ( $x = \varphi(n)$ )	<code>istotient(<math>x, \{\&amp;n\}</math>)</code>
is $x$ a polygonal number? ( $x = P(s, n)$ )	<code>ispolygonal(<math>x, s, \{\&amp;n\}</math>)</code>
is $pol$ irreducible?	<code>polisirreducible(<math>pol</math>)</code>

Graphic Functions

crude graph of $expr$ between $a$ and $b$	<code>plot(<math>X = a, b, expr</math>)</code>
High-resolution plot (immediate plot)	
plot $expr$ between $a$ and $b$	<code>plotoh(<math>X = a, b, expr, \{flag\}, \{n\}</math>)</code>
plot points given by lists $lx, ly$	<code>plotthraw(<math>lx, ly, \{flag\}</math>)</code>
terminal dimensions	<code>plotsizes()</code>

Rectwindow functions

init window $w$ , with size $x, y$	<code>plotinit(<math>w, x, y</math>)</code>
erase window $w$	<code>plotkill(<math>w</math>)</code>
copy $w$ to $w_2$ with offset $(dx, dy)$	<code>plotcopy(<math>w, w_2, dx, dy</math>)</code>
scale contents of $w$	<code>plotclip(<math>w</math>)</code>
scale coordinates in $w$	<code>plotscale(<math>w, x_1, x_2, y_1, y_2</math>)</code>
plotoh in $w$	<code>plotrecth(<math>w, X = a, b, expr, \{flag\}, \{n\}</math>)</code>
plotthraw in $w$	<code>plotrectthraw(<math>w, data, \{flag\}</math>)</code>
draw window $w_1$ at $(x_1, y_1), \dots$	<code>plotdraw(<math>[[w_1, x_1, y_1], \dots]</math>)</code>

Low-level Rectwindow Functions

set current drawing color in $w$ to $c$	<code>plotcolor(<math>w, c</math>)</code>
current position of cursor in $w$	<code>plotcursor(<math>w</math>)</code>
write $s$ at cursor's position	<code>plotstring(<math>w, s</math>)</code>
move cursor to $(x, y)$	<code>plotmove(<math>w, x, y</math>)</code>
move cursor to $(x + dx, y + dy)$	<code>plotrmove(<math>w, dx, dy</math>)</code>
draw a box to $(x_2, y_2)$	<code>plotbox(<math>w, x_2, y_2</math>)</code>
draw a box to $(x + dx, y + dy)$	<code>plotrbox(<math>w, dx, dy</math>)</code>
draw polygon	<code>plotlines(<math>w, lx, ly, \{flag\}</math>)</code>
draw points	<code>plotpoints(<math>w, lx, ly</math>)</code>
draw line to $(x + dx, y + dy)$	<code>plotrline(<math>w, dx, dy</math>)</code>
draw point $(x + dx, y + dy)$	<code>plotrpoint(<math>w, dx, dy</math>)</code>

Convert to Postscript or Scalable Vector Graphics

The format $f$ is either "ps" or "svg".	
as plotoh	<code>plotexport(<math>f, X = a, b, expr, \{flag\}, \{n\}</math>)</code>
as plotthraw	<code>plotthrawexport(<math>f, lx, ly, \{flag\}</math>)</code>
as plotdraw	<code>plotexport(<math>f, [[w_1, x_1, y_1], \dots]</math>)</code>