

Pari-GP reference card

(PARI-GP version 2.15.3)

Note: optional arguments are surrounded by braces {}.

To start the calculator, type its name in the terminal: **gp**

To exit **gp**, type **quit**, **\q**, or **<C-D>** at prompt.

Help

describe function	?function
extended description	??keyword
list of relevant help topics	???pattern
name of GP-1.39 function f in GP-2.*	whatnow(f)

Input/Output

previous result, the result before	%, %`, %`` , etc.
n -th result since startup	% n
separate multiple statements on line	;
extend statement on additional lines	\
extend statements on several lines	{ seq_1 ; seq_2 ;}
comment	/* ... */
one-line comment, rest of line ignored	\\ ...

Metacommands & Defaults

set default d to val	default({ d },{ val })
toggle timer on/off	#
print time for last result	##
print defaults	\d
set debug level to n	\g n
set memory debug level to n	\gm n
set n significant digits / bits	\p n , \pb n
set n terms in series	\ps n
quit GP	\q
print the list of PARI types	\t
print the list of user-defined functions	\u
read file into GP	\r <i>filename</i>
set debuglevel for domain D to n	setdebug(D,n)

Debugger / break loop

get out of break loop	break or <C-D>
go up/down n frames	dbg_up({ n }), dbg_down
set break point	breakpoint()
examine object o	dbg_x(o)
current error data	dbg_err()
number of objects on heap and their size	getheap()
total size of objects on PARI stack	getstack()

PARI Types & Input Formats

t_INT . Integers; hex, binary	± 31 ; $\pm 0x1F$, $\pm 0b101$
t_REAL . Reals	± 3.14 , 6.022 E23
t_INTMOD . Integers modulo m	Mod(n,m)
t_FRAC . Rational Numbers	n/m
t_FFELT . Elt in finite field \mathbf{F}_q	ffgen(q , 't)
t_COMPLEX . Complex Numbers	$x + y * I$
t_PADIC . p -adic Numbers	$x + O(p^k)$
t_QUAD . Quadratic Numbers	$x + y * \text{quadgen}(D, \{ 'w \})$
t_POLMOD . Polynomials modulo g	Mod(f,g)
t_POL . Polynomials	$a * x^n + \dots + b$
t_SER . Power Series	$f + O(x^k)$
t_RFRAC . Rational Functions	f/g
t_QFB . Binary quadratic form	Qfb(a,b,c)
t_VEC/t_COL . Row/Column Vectors	[x,y,z], [x,y,z]~
t_VEC integer range	[1..10]

t_VECSMALL . Vector of small ints	Vecsmall([x,y,z])
t_MAT . Matrices	[$a,b;c,d$]
t_LIST . Lists	List([x,y,z])
t_STR . Strings	"abc"
t_INFINITY . $\pm\infty$	+oo, -oo

Reserved Variable Names

$\pi \approx 3.14$, $\gamma \approx 0.57$, $C \approx 0.91$, $I = \sqrt{-1}$	Pi, Euler, Catalan, I
Landau's big-oh notation	O

Information about an Object, Precision

PARI type of object x	type(x)
length of x / size of x in memory	# x , sizebyte(x)
real precision / bit precision of x	precision(x), bitprecision(x)
p -adic, series prec. of x	padicprec(x,p), serprec(x,v)
current dynamic precision	getlocalprec, getlocalbitprec

Operators

basic operations	+, -, *, /, ^, sqr
$i \leftarrow i+1$, $i \leftarrow i-1$, $i \leftarrow i*j$, ...	i++, i--, i*=j,...
Euclidean quotient, remainder	$x \backslash y$, $x \backslash y$, $x \% y$, divrem(x,y)
shift x left or right n bits	$x << n$, $x >> n$ or shift($x, \pm n$)
multiply by 2^n	shiftmul(x,n)
comparison operators	<=, <, >=, >, ==, !=, ==, lex, cmp
boolean operators (or, and, not)	, &&, !
bit operations	bitand, bitneg, bitor, bitxor, bitnegimply
maximum/minimum of x and y	max(x,y), min(x,y)
sign of x (gives $-1, 0, 1$)	sign(x)
binary exponent of x	exponent(x)
derivative of f , 2nd derivative, etc.	f' , f'' , ...
differential operator	diffop($f,v,d,\{n=1\}$)
quote operator (formal variable)	'x
assignment	x = <i>value</i>
simultaneous assignment $x \leftarrow v[1]$, $y \leftarrow v[2]$	[x,y] = v

Select Components

Caveat: components start at index $n = 1$.

n -th component of x	component(x,n)
n -th component of vector/list x	$x[n]$
components $a, a+1, \dots, b$ of vector x	$x[a..b]$
(m,n) -th component of matrix x	$x[m,n]$
row m or column n of matrix x	$x[m,]$, $x[,n]$
numerator/denominator of x	numerator(x), denominator(x)

Random Numbers

random integer/prime in $[0,N[$	random(N), randomprime(N)
get/set random seed	getrand, setrand(s)

Conversions

to vector, matrix, vec. of small ints	Col/Vec, Mat, Vecsmall
to list, set, map, string	List, Set, Map, Str
create $(x \bmod y)$	Mod(x,y)
make x a polynomial of v	Pol($x,\{v\}$)
variants of Pol <i>et al.</i> , in reverse order	Polrev, Vecrev, Colrev
make x a power series of v	Ser($x,\{v\}$)
convert x to simplest possible type	simplify(x)
object x with real precision n	precision(x,n)
object x with bit precision n	bitprecision(x,n)
set precision to p digits in dynamic scope	localprec(p)
set precision to p bits in dynamic scope	localbitprec(p)

Character strings

convert to TeX representation	strtex(x)
string from bytes / from format+args	strchr, sprintf
split string / join strings	strsplit, strjoin
convert time t ms. to h, m, s, ms format	strtime(t)
Conjugates and Lifts	
conjugate of a number x	conj(x)
norm of x , product with conjugate	norm(x)
L^p norm of x (L^∞ if no p)	normlp($x,\{p\}$)
square of L^2 norm of x	norml2(x)
lift of x from Mods and p -adics	lift, centerlift(x)
recursive lift	liftall
lift all t_INT and t_PADIC (\rightarrow t_INT)	liftint
lift all t_POLMOD (\rightarrow t_POL)	liftpol

Lists, Sets & Maps

Sets (= row vector with strictly increasing entries w.r.t. **cmp**)

intersection of sets x and y	setintersect(x,y)
set of elements in x not belonging to y	setminus(x,y)
symmetric difference $x \Delta y$	setdelta(x,y)
union of sets x and y	setunion(x,y)
does y belong to the set x	setsearch($x,y,\{flag\}$)
set of all $f(x,y)$, $x \in X$, $y \in Y$	setbinop(f,X,Y)
is x a set ?	setisset(x)

Lists. create empty list: $L = \text{List}()$

append x to list L	listput($L,x,\{i\}$)
remove i -th component from list L	listpop($L,\{i\}$)
insert x in list L at position i	listinsert(L,x,i)
sort the list L in place	listsort($L,\{flag\}$)

Maps. create empty dictionary: $M = \text{Map}()$

attach value v to key k	mapput(M,k,v)
recover value attach to key k or error	mapget(M,k)
is key k in the dict? (set v to $M(k)$)	mapisdefined($M,k,\{\&v\}$)
remove k from map domain	mapdelete(M,k)

GP Programming

User functions and closures

x,y are formal parameters; y defaults to Pi if parameter omitted;
 z,t are local variables (lexical scope), z initialized to 1.

fun(x, y=Pi) = my(z=1, t); *seq*

fun = (x, y=Pi) -> my(z=1, t); *seq*

attach help message h to s	addhelp(s,h)
undefine symbol s (also kills help)	kill(s)
Control Statements (X : formal parameter in expression <i>seq</i>)	
if $a \neq 0$, evaluate seq_1 , else seq_2	if($a,\{seq_1\},\{seq_2\}$)
eval. <i>seq</i> for $a \leq X \leq b$	for($X = a,b,seq$)
...for $X \in v$	foreach(v,X,seq)
...for primes $a \leq X \leq b$	forprime($X = a,b,seq$)
...for primes $\equiv a \pmod q$	forprimestep($X = a,b,q,seq$)
...for composites $a \leq X \leq b$	forcomposite($X = a,b,seq$)
...for $a \leq X \leq b$ stepping s	forstep($X = a,b,s,seq$)
...for X dividing n	fordiv(n,X,seq)
... $X = [n, factor(n)]$, $a \leq n \leq b$	forfactored($X = a,b,seq$)
...as above, n squarefree	forsquarefree($X = a,b,seq$)
... $X = [d, factor(d)]$, $d \mid n$	fordivfactored(n,X,seq)
multivariable for , lex ordering	forvec($X = v,seq$)

```

loop over partitions of  $n$ 
... permutations of  $S$ 
... subsets of  $\{1, \dots, n\}$ 
...  $k$ -subsets of  $\{1, \dots, n\}$ 
... vectors  $v, q(v) \leq B$ ;  $q > 0$ 
...  $H < G$  finite abelian group
evaluate  $seq$  until  $a \neq 0$ 
while  $a \neq 0$ , evaluate  $seq$ 
exit  $n$  innermost enclosing loops
start new iteration of  $n$ -th enclosing loop
return  $x$  from current subroutine
Exceptions, warnings
raise an exception / warning
type of error message  $E$ 
try  $seq_1$ , evaluate  $seq_2$  on error
Functions with closure arguments / results
number of arguments of  $f$ 
select from  $v$  according to  $f$ 
apply  $f$  to all entries in  $v$ 
evaluate  $f(a_1, \dots, a_n)$ 
evaluate  $f(\dots f(f(a_1, a_2), a_3) \dots, a_n)$ 
calling function as closure
Sums & Products
sum  $X = a$  to  $X = b$ , initialized at  $x$ 
sum entries of vector  $v$ 
product of all vector entries
sum  $expr$  over divisors of  $n$ 
... assuming  $expr$  multiplicative
product  $a \leq X \leq b$ , initialized at  $x$ 
product over primes  $a \leq X \leq b$ 
Sorting
sort  $x$  by  $k$ -th component
min.  $m$  of  $x$  ( $m = x[i]$ ), max.
does  $y$  belong to  $x$ , sorted wrt.  $f$ 
 $\prod g^x \rightarrow$  factorization ( $\Rightarrow$  sorted, unique  $g$ )
Input/Output
print with/without  $\backslash n$ ,  $\text{\TeX}$  format
pretty print matrix
print fields with separator
formatted printing
write  $args$  to file
write  $x$  in binary format
read file into GP
... return as vector of lines
... return as vector of strings
read a string from keyboard
Files and file descriptors
File descriptors allow efficient small consecutive reads or writes
from or to a given file. The argument  $n$  below is always a descriptor,
attached to a file in r(ead), w(rite) or a(ppend) mode.
get descriptor  $n$  for file  $path$  in given  $mode$ 
... from shell  $cmd$  output (pipe)
close descriptor
commit pending write operations
read logical line from file
... raw line from file
write  $s \backslash n$  to file
... write  $s$  to file

```

```

forpart( $p = n, seq$ )
forperm( $S, p, seq$ )
forsubset( $n, p, seq$ )
forsubset( $[n, k], p, seq$ )
forqfvec( $v, q, b, seq$ )
forsubgroup( $H = G$ )
until( $a, seq$ )
while( $a, seq$ )
break( $\{n\}$ )
next( $\{n\}$ )
return( $\{x\}$ )

error(), warning()
errname( $E$ )
iferr( $seq_1, E, seq_2$ )

Results
arity( $f$ )
select( $f, v$ )
apply( $f, v$ )
call( $f, a$ )
fold( $f, a$ )
self()

sum( $X = a, b, expr, \{x\}$ )
vecsum( $v$ )
vecprod( $v$ )
sumdiv( $n, X, expr$ )
sumdivmult( $n, X, expr$ )
prod( $X = a, b, expr, \{x\}$ )
prodeuler( $X = a, b, expr$ )

vecsort( $x, \{k\}, \{fl = 0\}$ )
vecmin( $x, \{\&i\}$ ), vecmax
vecsearch( $x, y, \{f\}$ )
matreduce( $m$ )

print, print1, printtex
printp
printsep( $sep, \dots$ ), printsep1
printf()

write, write1, writetex( $file, args$ )
writebin( $file, x$ )
read( $\{file\}$ )
readvec( $\{file\}$ )
readstr( $\{file\}$ )
input()

fileclose( $n$ )
fileflush( $n$ )
fileread( $n$ )
filereadstr( $n$ )
filewrite( $n, s$ )
filewrite1( $n, s$ )

```

Pari-GP reference card

(PARI-GP version 2.15.3)

Timers

CPU time in ms and reset timer
CPU time in ms since gp startup
time in ms since UNIX Epoch
timeout command after s seconds

Interface with system

allocates a new stack of s bytes
alias old to new
install function from library
execute system command a
... and feed result to GP
... returning GP string
get \$VAR from environment
expand env. variable in string

```

gettime()
getabstime()
getwalltime()
alarm( $s, expr$ )

allocatemem( $\{s\}$ )
alias( $new, old$ )
install( $f, code, \{gpf\}, \{lib\}$ )
system( $a$ )
extern( $a$ )
externstr( $a$ )
getenv("VAR")
strexpand( $x$ )

```

Parallel evaluation

These functions evaluate their arguments in parallel (pthreads or MPI); args. must not access global variables (use **export** for this) and must be free of side effects. Enabled if threading engine is not *single* in gp header.

```

evaluate  $f$  on  $x[1], \dots, x[n]$ 
evaluate closures  $f[1], \dots, f[n]$ 
as select
as sum
as vector
eval  $f$  for  $i = a, \dots, b$ 
... for each element  $x$  in  $v$ 
... for  $p$  prime in  $[a, b]$ 
... for  $p = a \bmod q$ 
... multivariate
parforvec( $X = v, f, \{r\}, \{f_2\}, \{flag\}$ )
export  $x$  to parallel world
... all dynamic variables
frees exported value  $x$ 
... all exported values

```

```

parapply( $f, x$ )
pareval( $f$ )
parselect( $f, A, \{flag\}$ )
parsum( $i = a, b, expr$ )
parvector( $n, i, \{expr\}$ )
parfor( $i = a, \{b\}, f, \{r\}, \{f_2\}$ )
parforeach( $v, x, f, \{r\}, \{f_2\}$ )
parforprime( $p = a, \{b\}, f, \{r\}, \{f_2\}$ )
parforprimestep( $p = a, \{b\}, q, f, \{r\}, \{f_2\}$ )

```

Linear Algebra

dimensions of matrix x
multiply two matrices
... assuming result is diagonal
concatenation of x and y
extract components of x
transpose of vector or matrix x
adjoint of the matrix x
eigenvectors/values of matrix x
characteristic/minimal polynomial of x
trace/determinant of matrix x
permanent of matrix x
Frobenius form of x
QR decomposition
apply **matqr**'s transform to v

```

matsize( $x$ )
 $x * y$ 
matmultodiagonal( $x, y$ )
concat( $x, \{y\}$ )
vecextract( $x, y, \{z\}$ )
 $x \sim$ , mattranspose( $x$ )
matadjoint( $x$ )
mateigen( $x$ )
charpoly( $x$ ), minpoly( $x$ )
trace( $x$ ), matdet( $x$ )
matpermanent( $x$ )
matfrobenius( $x$ )
matqr( $x$ )
mathouseholder( $Q, v$ )

```

Constructors & Special Matrices

```

{ $g(x): x \in v$  s.t.  $f(x)$ }
{ $x: x \in v$  s.t.  $f(x)$ }
{ $g(x): x \in v$ }
row vec. of  $expr$  eval'ed at  $1 \leq i \leq n$ 
col. vec. of  $expr$  eval'ed at  $1 \leq i \leq n$ 
vector of small ints

```

```

[g(x) | x <- v, f(x)]
[x | x <- v, f(x)]
[g(x) | x <- v]
vector( $n, \{i\}, \{expr\}$ )
vectorv( $n, \{i\}, \{expr\}$ )
vectorsmall( $n, \{i\}, \{expr\}$ )

```

```

[ $c, c \cdot x, \dots, c \cdot x^n$ ]
[ $1, 2^x, \dots, n^x$ ]
matrix  $1 \leq i \leq m, 1 \leq j \leq n$ 
define matrix by blocks
diagonal matrix with diagonal  $x$ 
is  $x$  diagonal?
 $x \cdot \text{matdiagonal}(d)$ 
 $n \times n$  identity matrix
Hessenberg form of square matrix  $x$ 
 $n \times n$  Hilbert matrix  $H_{ij} = (i + j - 1)^{-1}$ 
 $n \times n$  Pascal triangle
companion matrix to polynomial  $x$ 
Sylvester matrix of  $x$  and  $y$ 

```

Gaussian elimination

```

kernel of matrix  $x$ 
intersection of column spaces of  $x$  and  $y$ 
solve  $MX = B$  ( $M$  invertible)
one sol of  $M * X = B$ 
basis for image of matrix  $x$ 
columns of  $x$  not in matimage
supplement columns of  $x$  to get basis
rows, cols to extract invertible matrix
rank of the matrix  $x$ 
solve  $MX = B \bmod D$ 
image mod  $D$ 
kernel mod  $D$ 
inverse mod  $D$ 
determinant mod  $D$ 

```

Lattices & Quadratic Forms

Quadratic forms

```

evaluate  ${}^t x Q y$ 
evaluate  ${}^t x Q x$ 
signature of quad form  ${}^t y * x * y$ 
decomp into squares of  ${}^t y * x * y$ 
eigenvalues/vectors for real symmetric  $x$ 

```

HNF and SNF

```

upper triangular Hermite Normal Form
HNF of  $x$  where  $d$  is a multiple of  $\det(x)$ 
multiple of  $\det(x)$ 
HNF of  $(x \mid \text{diagonal}(D))$ 
elementary divisors of  $x$ 
 $q$ -rank from elementary divisors
elementary divisors of  $\mathbf{Z}[a]/(f'(a))$ 
integer kernel of  $x$ 
 $\mathbf{Z}$ -module  $\leftrightarrow$   $\mathbf{Q}$ -vector space

```

Lattices

```

LLL-algorithm applied to columns of  $x$ 
... for Gram matrix of lattice
find up to  $m$  sols of qfnorm( $x, y) \leq b$ 
 $v, v[i] :=$  number of  $y$  s.t. qfnorm( $x, y) = i$ 
perfection rank of  $x$ 
find isomorphism between  $q$  and  $Q$ 
precompute for isomorphism test with  $q$ 
automorphism group of  $q$ 

```

Based on an earlier version by Joseph H. Silverman
November 2022 v2.38. Copyright © 2022 K. Belabas
Permission is granted to make and distribute copies of this card provided the
copyright and this permission notice are preserved on all copies.
Send comments and corrections to (Karim.Belabas@math.u-bordeaux.fr)

convert **qfauto** for GAP/Magma **qfautoexport**($G, \{flag\}$)
orbits of V under $G \subset \text{GL}(V)$ **qforbits**(G, V)

Polynomials & Rational Functions

all defined polynomial variables **variables**()
get var. of highest priority (higher than v) **varhigher**($name, \{v\}$)
... of lowest priority (lower than v) **varlower**($name, \{v\}$)

Coefficients, variables and basic operators

degree of f **poldegree**(f)
coef. of degree n of f , leading coef. **polcoef**(f, n), **pollead**
main variable / all variables in f **variable**(f), **variables**(f)
replace x by y in f **subst**(f, x, y)
evaluate f replacing vars by their value **eval**(f)
replace polynomial expr. $T(x)$ by y in f **substpol**(f, T, y)
replace x_1, \dots, x_n by y_1, \dots, y_n in f **substvec**(f, x, y)

$f \in A[x]$; reciprocal polynomial $x^{\deg f} f\left(\frac{1}{x}\right)$ **polrecip**(f)
gcd of coefficients of f **content**(f)
derivative of f w.r.t. x **deriv**($f, \{x\}$)
... n -th derivative of f **derivn**($f, n, \{x\}$)
formal integral of f w.r.t. x **intformal**($f, \{x\}$)
formal sum of f w.r.t. x **sumformal**($f, \{x\}$)

Constructors & Special Polynomials

interpolation polynomial at $(x[1], y[1]), \dots, (x[n], y[n])$, evaluated at t , with error estimate e **polinterpolate**($x, \{y\}, \{t\}, \{&e\}$)
 $T_n/U_n, H_n$ **polchebyshev**(n), **polhermite**(n)
 $P_n, L_n^{(\alpha)}$ **pollegendre**(n), **pollaguerre**(n, a)
 n -th cyclotomic polynomial Φ_n **polcyclo**(n)
return n if $f = \Phi_n$, else 0 **poliscyclo**(f)
is f a product of cyclotomic polynomials? **poliscycloprod**(f)
Zagier's polynomial of index (n, m) **polzagier**(n, m)

Resultant, elimination

discriminant of polynomial f **poldisc**(f)
find factors of **poldisc**(f) **poldiscfactors**(f)
resultant $R = \text{Res}_v(f, g)$ **polresultant**($f, g, \{v\}$)
 $[u, v, R], xu + yv = \text{Res}_v(f, g)$ **polresultanttext**($x, y, \{v\}$)
solve Thue equation $f(x, y) = a$ **thue**($t, a, \{sol\}$)
initialize t for Thue equation solver **thueinit**(f)

Roots and Factorization (Complex/Real)

complex roots of f **polroots**(f)
bound complex roots of f **polrootsbound**(f)
number of real roots of f (in $[a, b]$) **polsturm**($f, \{[a, b]\}$)
real roots of f (in $[a, b]$) **polrootsreal**($f, \{[a, b]\}$)
complex embeddings of **t_POLMOD** z **conjvec**(z)

Roots and Factorization (Finite fields)

factor f mod p , roots **factormod**(f, p), **polrootsmod**
factor f over $\mathbf{F}_p[x]/(T)$, roots **factormod**($f, [T, p]$), **polrootsmod**
squarefree factorization of f in $\mathbf{F}_q[x]$ **factormodSQF**($f, \{D\}$)
distinct degree factorization of f in $\mathbf{F}_q[x]$ **factormodDDF**($f, \{D\}$)
factor n -th cyclotomic pol. Φ_n mod p **factormodcyclo**(n, p)

Roots and Factorization (p -adic fields)

factor f over \mathbf{Q}_p , roots **factorpadic**(f, p, r), **polrootspadic**
 p -adic root of f congruent to a mod p **padicappr**(f, a)
Newton polygon of f for prime p **newtonpoly**(f, p)
Hensel lift $A/\text{lc}(A) = \prod_i B[i]$ mod p^e **polhensellift**(A, B, p, e)
 $T = \prod (x - z_i) \mapsto \prod [x - \omega(z_i)] \in \mathbf{Z}_p[x]$ **polteichmuller**(T, p, e)
extensions of \mathbf{Q}_p of degree N **padicfields**(p, N)

Pari-GP reference card

(PARI-GP version 2.15.3)

Roots and Factorization (Miscellaneous)

symmetric powers of roots of f up to n **polsym**(f, n)
Graeffe transform of $f, g(x^2) = f(x)f(-x)$ **polgraeffe**(f)
factor f over coefficient field **factor**(f)
cyclotomic factors of $f \in \mathbf{Q}[X]$ **polcyclofactors**(f)

Finite Fields

A finite field is encoded by any element (**t_FFELT**).
find irreducible $T \in \mathbf{F}_p[x]$, $\deg T = n$ **ffinit**($p, n, \{x\}$)
Create t in $\mathbf{F}_q \simeq \mathbf{F}_p[t]/(T)$ **t = ffg**($T, 't$)
... indirectly, with implicit T **t = ffg**($q, 't$); **T = t.mod**
map m from $\mathbf{F}_q \ni a$ to $\mathbf{F}_{q^k} \ni b$ **m = ffg**(a, b)
build $K = \mathbf{F}_q[x]/(P)$ extending $\mathbf{F}_q \ni a$, **ffextend**(a, P)
evaluate map m on x **ffmap**(m, x)
inverse map of m **ffinvmap**(m)
compose maps $m \circ n$ **ffcompomap**(m, n)
 x as polmod over codomain of map m **ffmaprel**(m, x)
 F^n over $\mathbf{F}_q \ni a$ **fffrobenius**(a, n)
 $\#$ {monic irred. $T \in \mathbf{F}_q[x]$, $\deg T = n$ } **ffnbirred**(q, n)

Formal & p -adic Series

truncate power series or p -adic number **truncate**(x)
valuation of x at p **valuation**(x, p)
Dirichlet and Power Series
Taylor expansion around 0 of f w.r.t. x **taylor**(f, x)
Laurent series of closure F up to x^k **laurentseries**(f, k)
 $\sum a_k b_k t^k$ from $\sum a_k t^k$ and $\sum b_k t^k$ **serconvol**(a, b)
 $f = \sum a_k t^k$ from $\sum (a_k/k!) t^k$ **serlaplace**(f)
reverse power series F so $F(f(x)) = x$ **serreverse**(f)
remove terms of degree $< n$ in f **serchop**(f, n)
Dirichlet series multiplication / division **dirmul, dirdiv**(x, y)
Dirichlet Euler product (b terms) **direuler**($p = a, b, expr$)

Transcendental and p -adic Functions

real, imaginary part of x **real**(x), **imag**(x)
absolute value, argument of x **abs**(x), **arg**(x)
square/nth root of x **sqrt**(x), **sqrtn**($x, n, \{&z\}$)
all n -th roots of 1 **rootsof1**(n)
FFT of $[f_0, \dots, f_{n-1}]$ **w = fftinit**(n), **fft/fftin**(w, f)
trig functions **sin, cos, tan, cotan, sinc**
inverse trig functions **asin, acos, atan**
hyperbolic functions **sinh, cosh, tanh, cotanh**
inverse hyperbolic functions **asinh, acosh, atanh**
 $\log(x), \log(1+x), e^x, e^x - 1$ **log, loglp, exp, expm1**
Euler Γ function, $\log \Gamma, \Gamma'/\Gamma$ **gamma, lngamma, psi**
half-integer gamma function $\Gamma(n+1/2)$ **gammah**(n)
Riemann's zeta $\zeta(s) = \sum n^{-s}$ **zeta**(s)
 $\sum_{1 \leq n \leq N} n^s$ **dirpowerssum**(N, s)
Hurwitz's $\zeta(s, x) = \sum (n+x)^{-s}$ **zetahurwitz**(s, x)
Lerch $\Phi(z, s, x) = \sum z^n (n+x)^{-s}$ **lerchphi**(z, s, x)
Lerch $L(s, x, t) = \Phi(e^{2i\pi t}, s, x)$ **lerchzeta**(s, x, t)
multiple zeta value (MZV), $\zeta(s_1, \dots, s_k)$ **zetamult**($s, \{T\}$)
all MZVs for weight $\sum s_i = n$ **zetamultall**(n)
convert MZV id to $[s_1, \dots, s_k]$ **zetamultconvert**($f, \{flag\}$)
MZV dual sequence **zetamultdual**(s)
multiple polylog $Li_{s_1, \dots, s_k}(z_1, \dots, z_k)$ **polylogmult**(s, z)

incomplete Γ function ($y = \Gamma(s)$) **incgam**($s, x, \{y\}$)
complementary incomplete Γ **incgamc**(s, x)
 $\int_x^\infty e^{-t} dt/t, (2/\sqrt{\pi}) \int_x^\infty e^{-t^2} dt$ **eint1, erfc**
elliptic integral of 1st and 2nd kind **ellK**(k), **ellE**(k)
dilogarithm of x **dilog**(x)
 m -th polylogarithm of x **polylog**($m, x, \{flag\}$)
 U -confluent hypergeometric function **hyperu**(a, b, u)
Hypergeometric ${}_pF_q(A, B; z)$ **hypergeom**(A, B, z)
Bessel $J_n(x), J_{n+1/2}(x)$ **besselj**(n, x), **besseljh**(n, x)
Bessel $I_\nu, K_\nu, H_\nu^1, H_\nu^2, Y_\nu$ **(bessel)i, k, h1, h2, y**
 k -th zero of $J_\nu(x)$ **besseljzero**($nu, \{k=1\}$)
 k -th zero of $Y_\nu(x)$ **besselyzero**($nu, \{k=1\}$)
Airy functions $A_i(x), B_i(x)$ **airy**(x)
Lambert $W: x$ s.t. $xe^x = y$ **lambertw**(y)
Teichmuller character of p -adic x **teichmuller**(x)

Iterations, Sums & Products

Numerical integration for meromorphic functions

Behaviour at endpoint for Double Exponential (DE) methods: either a scalar ($a \in \mathbf{C}$, regular) or $\pm\infty$ (decreasing at least as x^{-2}) or
 $(x-a)^{-\alpha}$ singularity **[a, a]**
exponential decrease $e^{-\alpha|x|}$ **[$\pm\infty, a$], $\alpha > 0$**
slow decrease $|x|^\alpha$ **$\dots \alpha < -1$**
oscillating as $\cos(kx)$ **$\alpha = k\mathbf{I}, k > 0$**
oscillating as $\sin(kx)$ **$\alpha = -k\mathbf{I}, k > 0$**

numerical integration **intnum**($x = a, b, f, \{T\}$)
weights T for **intnum** **intnuminit**($a, b, \{m\}$)
weights T incl. kernel K **intfuncinit**($t = a, b, K, \{m\}$)
integrate $(2i\pi)^{-1} f$ on circle $|z-a| = R$ **intcirc**($x = a, R, f, \{T\}$)
Other integration methods
 n -point Gauss-Legendre **intnumgauss**($x = a, b, f, \{n\}$)
weights for n -point Gauss-Legendre **intnumgaussinit**($\{n\}$)
quasi-periodic function, period $2H$ **intnumosc**($x = a, f, H$)
Romberg (low accuracy) **intnumromb**($x = a, b, f, \{flag\}$)

Numerical summation

sum of series $f(n), n \geq a$ (low accuracy) **suminf**($n = a, expr$)
sum of alternating/positive series **sumalt, sumpos**
sum of series using Euler-Maclaurin **sumnum**($n = a, f, \{T\}$)
... Sidi summation **sumnumsidi**($n = a, f$)
 $\sum_{n \geq a} F(n)$, F rational function **sumnumrat**(F, a)
 $\dots \sum_{p \geq a} F(p^s)$ **sumeulerrat**($F, \{s=1\}, \{a=2\}$)
weights for **sumnum**, a as in DE **sumnuminit**($\{\infty, a\}$)
sum of series by Monien summation **sumnummonien**($n = a, f, \{T\}$)
weights for **sumnummonien** **sumnummonieninit**($\{\infty, a\}$)
sum of series using Abel-Plana **sumnumap**($n = a, f, \{T\}$)
weights for **sumnumap**, a as in DE **sumnumapinit**($\{\infty, a\}$)
sum of series using Lagrange **sumnumlagrange**($n = a, f, \{T\}$)
weights for **sumnumlagrange** **sumnumlagrangeinit**

Products

product $a \leq X \leq b$, initialized at x **prod**($X = a, b, expr, \{x\}$)
product over primes $a \leq X \leq b$ **prodeuler**($X = a, b, expr$)
infinite product $a \leq X \leq \infty$ **prodinf**($X = a, expr$)
 $\prod_{n \geq a} F(n)$, F rational function **prodnumrat**(F, a)
 $\prod_{p \geq a} F(p^s)$ **prodeulerrat**($F, \{s=1\}, \{a=2\}$)

Other numerical methods

real root of f in $[a, b]$; bracketed root	<code>solve($X = a, b, f$)</code>
...interval splitting, step s	<code>solvestep($X = a, b, s, f, \{flag = 0\}$)</code>
limit of $f(t)$, $t \rightarrow \infty$	<code>limitnum($f, \{\alpha\}$)</code>
asymptotic expansion of f (rational)	<code>asypnum($f, \{\alpha\}$)</code>
... $N + 1$ terms as floats	<code>asypnumraw($f, N, \{\alpha\}$)</code>
numerical derivation w.r.t x : $f'(a)$	<code>derivnum($x = a, f$)</code>
evaluate continued fraction F at t	<code>contfraceval($F, t, \{L\}$)</code>
power series to cont. fraction (L terms)	<code>contfracinit($S, \{L\}$)</code>
Padé approximant (deg. denom. $\leq B$)	<code>bestapprPade($S, \{B\}$)</code>

Elementary Arithmetic Functions

vector of binary digits of $ x $	<code>binary(x)</code>
bit number n of integer x	<code>bittest(x, n)</code>
Hamming weight of integer x	<code>hammingweight(x)</code>
digits of integer x in base B	<code>digits($x, \{B = 10\}$)</code>
sum of digits of integer x in base B	<code>sumdigits($x, \{B = 10\}$)</code>
integer from digits	<code>fromdigits($v, \{B = 10\}$)</code>
ceiling/floor/fractional part	<code>ceil, floor, frac</code>
round x to nearest integer	<code>round($x, \{\&e\}$)</code>
truncate x	<code>truncate($x, \{\&e\}$)</code>
gcd/LCM of x and y	<code>gcd(x, y), lcm(x, y)</code>
gcd of entries of a vector/matrix	<code>content(x)</code>

Primes and Factorization

extra prime table	<code>addprimes()</code>
add primes in v to prime table	<code>addprimes(v)</code>
remove primes from prime table	<code>removeprimes(v)</code>
Chebyshev $\pi(x)$, n -th prime p_n	<code>primepi(x), prime(n)</code>
vector of first n primes	<code>primes(n)</code>
smallest prime $\geq x$	<code>nextprime(x)</code>
largest prime $\leq x$	<code>precprime(x)</code>
factorization of x	<code>factor($x, \{lim\}$)</code>
...selecting specific algorithms	<code>factorint($x, \{flag = 0\}$)</code>
$n = df^2$, d squarefree/fundamental	<code>core($n, \{fl\}$), coredisc</code>
certificate for (prime) N	<code>primecert(N)</code>
verifies a certificate c	<code>primecertisvalid(c)</code>
convert certificate to Magma/PRIMO	<code>primecertexport</code>
recover x from its factorization	<code>factorback($f, \{e\}$)</code>
$x \in \mathbf{Z}$, $ x \leq X$, $\gcd(N, P(x)) \geq N$	<code>zncoppersmith($P, N, X, \{B\}$)</code>
divisors of N in residue class r mod s	<code>divisorslensstra(N, r, s)</code>

Divisors and multiplicative functions

number of prime divisors $\omega(n)$ / $\Omega(n)$	<code>omega(n), bigomega</code>
divisors of n / number of divisors $\tau(n)$	<code>divisors(n), numdiv</code>
sum of (k -th powers of) divisors of n	<code>sigma($n, \{k\}$)</code>
Möbius μ -function	<code>moebius(x)</code>
Ramanujan's τ -function	<code>ramanujantau(x)</code>

Combinatorics

factorial of x	<code>x!</code> or <code>factorial(x)</code>
binomial coefficient $\binom{x}{k}$	<code>binomial($x, \{k\}$)</code>
Bernoulli number B_n as real/rational	<code>bernreal(n), bernfrac</code>
$[B_0, B_2, \dots B_{2k}]$	<code>bernvec(k)</code>
Bernoulli polynomial $B_n(x)$	<code>bernpol($n, \{x\}$)</code>
Euler numbers	<code>eulerfrac, eulerreal, eulervec</code>
Euler polynomial $E_n(x)$	<code>eulerpol($n, \{x\}$)</code>
Eulerian polynomial $A_n(x)$	<code>eulerianpol</code>
Fibonacci number F_n	<code>fibonacci(n)</code>
Harmonic number $H_{n,r} = 1^{-r} + \dots + n^{-r}$	<code>harmonic(n, r)</code>
Stirling numbers $s(n, k)$ and $S(n, k)$	<code>stirling($n, k, \{flag\}$)</code>

Pari-GP reference card

(PARI-GP version 2.15.3)

number of partitions of n	<code>numbpart(n)</code>
k -th permutation on n letters	<code>numtoperm(n, k)</code>
...index k of permutation v	<code>permtounum(v)</code>
order of permutation p	<code>permorder(p)</code>
signature of permutation p	<code>permsign(p)</code>
cyclic decomposition of permutation p	<code>permcycles(p)</code>

Multiplicative groups $(\mathbf{Z}/N\mathbf{Z})^*$, \mathbf{F}_q^*

Euler ϕ -function	<code>eulerphi(x)</code>
multiplicative order of x (divides ϕ)	<code>znorder($x, \{o\}$), fforder</code>
primitive root mod q / x .mod	<code>znprimroot(q), fprimroot(x)</code>
structure of $(\mathbf{Z}/n\mathbf{Z})^*$	<code>znstar(n)</code>
discrete logarithm of x in base g	<code>znlog($x, g, \{o\}$), fflag</code>
Kronecker-Legendre symbol $(\frac{x}{y})$	<code>kronecker(x, y)</code>
quadratic Hilbert symbol (at p)	<code>hilbert($x, y, \{p\}$)</code>

Euclidean algorithm, continued fractions

CRT: solve $z \equiv x$ and $z \equiv y$	<code>chinese(x, y)</code>
minimal u, v so $xu + yv = \gcd(x, y)$	<code>gcdext(x, y)</code>
half-gcd algorithm	<code>halfgcd(x, y)</code>
continued fraction of x	<code>confrac($x, \{b\}, \{lmax\}$)</code>
last convergent of continued fraction x	<code>confracpnqn(x)</code>
rational approximation to x (den. $\leq B$)	<code>bestappr($x, \{B\}$)</code>
recognize $x \in \mathbf{C}$ as polmod mod $T \in \mathbf{Z}[X]$	<code>bestapprnf(x, T)</code>

Miscellaneous

integer square / n -th root of x	<code>sqrtint(x), sqtrtnint(x, n)</code>
largest integer e s.t. $b^e \leq b$, $e = \lfloor \log_b(x) \rfloor$	<code>logint($x, b, \{\&z\}$)</code>

Characters

Let $cyc = [d_1, \dots, d_k]$ represent an abelian group $G = \oplus (\mathbf{Z}/d_j\mathbf{Z}) \cdot g_j$ or any structure G affording a .cyc method; e.g. `znstar($q, 1$)` for Dirichlet characters. A character χ is coded by $[c_1, \dots, c_k]$ such that $\chi(g_j) = e(n_j/d_j)$.
 $\chi \cdot \psi$; χ^{-1} ; $\chi \cdot \psi^{-1}$; χ^k `charmul, charconj, chardiv, charpow`
order of χ `charorder(cyc, χ)`
kernel of χ `charker(cyc, χ)`
 $\chi(x)$, G a GP group structure `chareval($G, \chi, x, \{z\}$)`
Galois orbits of characters `chargalois(G)`

Dirichlet Characters

initialize $G = (\mathbf{Z}/q\mathbf{Z})^*$	<code>G = znstar($q, 1$)</code>
convert datum D to $[G, \chi]$	<code>znchar(D)</code>
is χ odd?	<code>zncharisodd(G, χ)</code>
real $\chi \rightarrow$ Kronecker symbol (D/\cdot)	<code>znchartokronecker(G, χ)</code>
conductor of χ	<code>zncharconductor(G, χ)</code>
$[G_0, \chi_0]$ primitive attached to χ	<code>znchartoprimitive(G, χ)</code>
induce $\chi \in \hat{G}$ to $\mathbf{Z}/N\mathbf{Z}$	<code>zncharinduce(G, χ, N)</code>
χp	<code>znchardecompose(G, χ, p)</code>
$\prod_p (\chi, N) \chi p$	<code>znchardecompose(G, χ, Q)</code>
complex Gauss sum $G_a(\chi)$	<code>znchargauss(G, χ)</code>

Conrey labelling

Conrey label $m \in (\mathbf{Z}/q\mathbf{Z})^* \rightarrow$ character	<code>znconreychar(G, m)</code>
character \rightarrow Conrey label	<code>znconreyexp(G, χ)</code>
log on Conrey generators	<code>znconreylog(G, m)</code>
conductor of χ (χ_0 primitive)	<code>znconreyconductor($G, \chi, \{\chi_0\}$)</code>

True-False Tests

is x the disc. of a quadratic field?	<code>isfundamental(x)</code>
is x a prime?	<code>isprime(x)</code>
is x a strong pseudo-prime?	<code>ispseudoprime(x)</code>
is x square-free?	<code>issquarefree(x)</code>
is x a square?	<code>issquare($x, \{\&n\}$)</code>
is x a perfect power?	<code>ispower($x, \{k\}, \{\&n\}$)</code>
is x a perfect power of a prime? ($x = p^n$)	<code>isprimepower($x, \&n$)</code>
... of a pseudoprime?	<code>ispseudoprimepower($x, \&n$)</code>
is x powerful?	<code>ispowerful(x)</code>
is x a totient? ($x = \varphi(n)$)	<code>istotient($x, \{\&n\}$)</code>
is x a polygonal number? ($x = P(s, n)$)	<code>ispolygonal($x, s, \{\&n\}$)</code>
is pol irreducible?	<code>polisirreducible(pol)</code>

Graphic Functions

crude graph of $expr$ between a and b	<code>plot($X = a, b, expr$)</code>
High-resolution plot (immediate plot)	
plot $expr$ between a and b	<code>ploth($X = a, b, expr, \{flag\}, \{n\}$)</code>
plot points given by lists lx, ly	<code>plothraw($lx, ly, \{flag\}$)</code>
terminal dimensions	<code>plotsizes()</code>

Rectwindow functions

init window w , with size x, y	<code>plotinit(w, x, y)</code>
erase window w	<code>plotkill(w)</code>
copy w to w_2 with offset (dx, dy)	<code>plotcopy(w, w_2, dx, dy)</code>
slice contents of w	<code>plotclip(w)</code>
scale coordinates in w	<code>plotscale(w, x_1, x_2, y_1, y_2)</code>
ploth in w	<code>plotrecth($w, X = a, b, expr, \{flag\}, \{n\}$)</code>
plothraw in w	<code>plotrecthraw($w, data, \{flag\}$)</code>
draw window w_1 at $(x_1, y_1), \dots$	<code>plotdraw($[[w_1, x_1, y_1], \dots]$)</code>

Low-level Rectwindow Functions

set current drawing color in w to c	<code>plotcolor(w, c)</code>
current position of cursor in w	<code>plotcursor(w)</code>
write s at cursor's position	<code>plotstring(w, s)</code>
move cursor to (x, y)	<code>plotmove(w, x, y)</code>
move cursor to $(x + dx, y + dy)$	<code>plotrmove(w, dx, dy)</code>
draw a box to (x_2, y_2)	<code>plotbox(w, x_2, y_2)</code>
draw a box to $(x + dx, y + dy)$	<code>plotrbox(w, dx, dy)</code>
draw polygon	<code>plotlines($w, lx, ly, \{flag\}$)</code>
draw points	<code>plotpoints(w, lx, ly)</code>
draw line to $(x + dx, y + dy)$	<code>plotrline(w, dx, dy)</code>
draw point $(x + dx, y + dy)$	<code>plotrpoint(w, dx, dy)</code>

Convert to Postscript or Scalable Vector Graphics

The format f is either "ps" or "svg".	
as ploth	<code>plotlexport($f, X = a, b, expr, \{flag\}, \{n\}$)</code>
as plothraw	<code>plotlrawexport($f, lx, ly, \{flag\}$)</code>
as plotdraw	<code>plotexport($f, [[w_1, x_1, y_1], \dots]$)</code>

Based on an earlier version by Joseph H. Silverman
November 2022 v2.38. Copyright © 2022 K. Belabas
Permission is granted to make and distribute copies of this card provided the copyright and this permission notice are preserved on all copies.
Send comments and corrections to (Karim.Belabas@math.u-bordeaux.fr)