

Pari-GP reference card

(PARI-GP version 2.15.3)

Note: optional arguments are surrounded by braces {}.

To start the calculator, type its name in the terminal: **gp**

To exit **gp**, type **quit**, **\q**, or **<C-D>** at prompt.

Help

| | |
|--|----------------|
| describe function | ?function |
| extended description | ??keyword |
| list of relevant help topics | ???pattern |
| name of GP-1.39 function f in GP-2.* | whatnow(f) |

Input/Output

| | |
|--|------------------------|
| previous result, the result before | %, %`, %`` , etc. |
| n -th result since startup | % n |
| separate multiple statements on line | ; |
| extend statement on additional lines | \ |
| extend statements on several lines | { seq_1 ; seq_2 ;} |
| comment | /* ... */ |
| one-line comment, rest of line ignored | \\ ... |

Metacommands & Defaults

| | |
|--|----------------------------|
| set default d to val | default({ d },{ val }) |
| toggle timer on/off | # |
| print time for last result | ## |
| print defaults | \d |
| set debug level to n | \g n |
| set memory debug level to n | \gm n |
| set n significant digits / bits | \p n , \pb n |
| set n terms in series | \ps n |
| quit GP | \q |
| print the list of PARI types | \t |
| print the list of user-defined functions | \u |
| read file into GP | \r $filename$ |
| set debuglevel for domain D to n | setdebug(D,n) |

Debugger / break loop

| | |
|--|---------------------------|
| get out of break loop | break or <C-D> |
| go up/down n frames | dbg_up({ n }), dbg_down |
| set break point | breakpoint() |
| examine object o | dbg_x(o) |
| current error data | dbg_err() |
| number of objects on heap and their size | getheap() |
| total size of objects on PARI stack | getstack() |

PARI Types & Input Formats

| | |
|---|---------------------------------------|
| t_INT . Integers; hex, binary | ± 31 ; $\pm 0x1F$, $\pm 0b101$ |
| t_REAL . Reals | ± 3.14 , 6.022 E23 |
| t_INTMOD . Integers modulo m | Mod(n,m) |
| t_FRAC . Rational Numbers | n/m |
| t_FFELT . Elt in finite field \mathbf{F}_q | ffgen(q , 't) |
| t_COMPLEX . Complex Numbers | $x + y * I$ |
| t_PADIC . p -adic Numbers | $x + O(p^k)$ |
| t_QUAD . Quadratic Numbers | $x + y * \text{quadgen}(D, \{ 'w \})$ |
| t_POLMOD . Polynomials modulo g | Mod(f,g) |
| t_POL . Polynomials | $a * x^n + \dots + b$ |
| t_SER . Power Series | $f + O(x^k)$ |
| t_RFRAC . Rational Functions | f/g |
| t_QFB . Binary quadratic form | Qfb(a,b,c) |
| t_VEC/t_COL . Row/Column Vectors | [x,y,z], [x,y,z]~ |
| t_VEC integer range | [1..10] |

| | |
|--|-----------------------|
| t_VECSMALL . Vector of small ints | Vecsmall([x,y,z]) |
| t_MAT . Matrices | [$a,b;c,d$] |
| t_LIST . Lists | List([x,y,z]) |
| t_STR . Strings | "abc" |
| t_INFINITY . $\pm\infty$ | +oo, -oo |

Reserved Variable Names

| | |
|---|-----------------------|
| $\pi \approx 3.14$, $\gamma \approx 0.57$, $C \approx 0.91$, $I = \sqrt{-1}$ | Pi, Euler, Catalan, I |
| Landau's big-oh notation | O |

Information about an Object, Precision

| | |
|---------------------------------------|---------------------------------------|
| PARI type of object x | type(x) |
| length of x / size of x in memory | # x , sizebyte(x) |
| real precision / bit precision of x | precision(x), bitprecision(x) |
| p -adic, series prec. of x | padicprec(x,p), serprec(x,v) |
| current dynamic precision | getlocalprec, getlocalbitprec |

Operators

| | |
|--|--|
| basic operations | +, -, *, /, ^, sqr |
| $i \leftarrow i+1$, $i \leftarrow i-1$, $i \leftarrow i*j$, ... | i++, i--, i*=j,... |
| Euclidean quotient, remainder | $x \backslash y$, $x \backslash y$, $x \% y$, divrem(x,y) |
| shift x left or right n bits | $x << n$, $x >> n$ or shift($x, \pm n$) |
| multiply by 2^n | shiftmul(x,n) |
| comparison operators | <=, <, >=, >, ==, !=, ==, lex, cmp |
| boolean operators (or, and, not) | , &&, ! |
| bit operations | bitand, bitneg, bitor, bitxor, bitnegimply |
| maximum/minimum of x and y | max(x,y), min(x,y) |
| sign of x (gives $-1, 0, 1$) | sign(x) |
| binary exponent of x | exponent(x) |
| derivative of f , 2nd derivative, etc. | f' , f'' , ... |
| differential operator | diffop($f,v,d,\{n=1\}$) |
| quote operator (formal variable) | 'x |
| assignment | x = value |
| simultaneous assignment $x \leftarrow v[1]$, $y \leftarrow v[2]$ | [x,y] = v |

Select Components

| | |
|---|--------------------------------------|
| <i>Caveat</i> : components start at index $n = 1$. | |
| n -th component of x | component(x,n) |
| n -th component of vector/list x | $x[n]$ |
| components $a, a+1, \dots, b$ of vector x | $x[a..b]$ |
| (m,n) -th component of matrix x | $x[m,n]$ |
| row m or column n of matrix x | $x[m,]$, $x[,n]$ |
| numerator/denominator of x | numerator(x), denominator(x) |

Random Numbers

| | |
|---------------------------------|-----------------------------------|
| random integer/prime in $[0,N[$ | random(N), randomprime(N) |
| get/set random seed | getrand, setrand(s) |

Conversions

| | |
|--|------------------------|
| to vector, matrix, vec. of small ints | Col/Vec, Mat, Vecsmall |
| to list, set, map, string | List, Set, Map, Str |
| create $(x \bmod y)$ | Mod(x,y) |
| make x a polynomial of v | Pol($x,\{v\}$) |
| variants of Pol <i>et al.</i> , in reverse order | Polrev, Vecrev, Colrev |
| make x a power series of v | Ser($x,\{v\}$) |
| convert x to simplest possible type | simplify(x) |
| object x with real precision n | precision(x,n) |
| object x with bit precision n | bitprecision(x,n) |
| set precision to p digits in dynamic scope | localprec(p) |
| set precision to p bits in dynamic scope | localbitprec(p) |

Character strings

| | |
|---|-------------------------|
| convert to TeX representation | strtex(x) |
| string from bytes / from format+args | strchr, sprintf |
| split string / join strings | strsplit, strjoin |
| convert time t ms. to h, m, s, ms format | strtime(t) |
| Conjugates and Lifts | |
| conjugate of a number x | conj(x) |
| norm of x , product with conjugate | norm(x) |
| L^p norm of x (L^∞ if no p) | normlp($x,\{p\}$) |
| square of L^2 norm of x | norml2(x) |
| lift of x from Mods and p -adics | lift, centerlift(x) |
| recursive lift | liftall |
| lift all t_INT and t_PADIC (\rightarrow t_INT) | liftint |
| lift all t_POLMOD (\rightarrow t_POL) | liftpol |

Lists, Sets & Maps

| | |
|--|-----------------------------|
| Sets (= row vector with strictly increasing entries w.r.t. cmp) | |
| intersection of sets x and y | setintersect(x,y) |
| set of elements in x not belonging to y | setminus(x,y) |
| symmetric difference $x \Delta y$ | setdelta(x,y) |
| union of sets x and y | setunion(x,y) |
| does y belong to the set x | setsearch($x,y,\{flag\}$) |
| set of all $f(x,y)$, $x \in X$, $y \in Y$ | setbinop(f,X,Y) |
| is x a set ? | setisset(x) |

| | |
|---|-------------------------------|
| Lists . create empty list: $L = \text{List}()$ | |
| append x to list L | listput($L,x,\{i\}$) |
| remove i -th component from list L | listpop($L,\{i\}$) |
| insert x in list L at position i | listinsert(L,x,i) |
| sort the list L in place | listsort($L,\{flag\}$) |
| Maps . create empty dictionary: $M = \text{Map}()$ | |
| attach value v to key k | mapput(M,k,v) |
| recover value attach to key k or error | mapget(M,k) |
| is key k in the dict? (set v to $M(k)$) | mapisdefined($M,k,\{\&v\}$) |
| remove k from map domain | mapdelete(M,k) |

GP Programming

User functions and closures

x,y are formal parameters; y defaults to Pi if parameter omitted; z,t are local variables (lexical scope), z initialized to 1.

fun(x, y=Pi) = my(z=1, t); seq

fun = (x, y=Pi) -> my(z=1, t); seq

| | |
|--|---------------------------------|
| attach help message h to s | addhelp(s,h) |
| undefine symbol s (also kills help) | kill(s) |
| Control Statements (X : formal parameter in expression seq) | |
| if $a \neq 0$, evaluate seq_1 , else seq_2 | if($a,\{seq_1\},\{seq_2\}$) |
| eval. seq for $a \leq X \leq b$ | for($X = a,b,seq$) |
| ...for $X \in v$ | foreach(v,X,seq) |
| ...for primes $a \leq X \leq b$ | forprime($X = a,b,seq$) |
| ...for primes $\equiv a \pmod q$ | forprimestep($X = a,b,q,seq$) |
| ...for composites $a \leq X \leq b$ | forcomposite($X = a,b,seq$) |
| ...for $a \leq X \leq b$ stepping s | forstep($X = a,b,s,seq$) |
| ...for X dividing n | fordiv(n,X,seq) |
| ... $X = [n, factor(n)]$, $a \leq n \leq b$ | forfactored($X = a,b,seq$) |
| ...as above, n squarefree | forsquarefree($X = a,b,seq$) |
| ... $X = [d, factor(d)]$, $d n$ | fordivfactored(n,X,seq) |
| multivariable for, lex ordering | forvec($X = v,seq$) |

```

loop over partitions of  $n$ 
... permutations of  $S$ 
... subsets of  $\{1, \dots, n\}$ 
...  $k$ -subsets of  $\{1, \dots, n\}$ 
... vectors  $v, q(v) \leq B$ ;  $q > 0$ 
...  $H < G$  finite abelian group
evaluate  $seq$  until  $a \neq 0$ 
while  $a \neq 0$ , evaluate  $seq$ 
exit  $n$  innermost enclosing loops
start new iteration of  $n$ -th enclosing loop
return  $x$  from current subroutine
Exceptions, warnings
raise an exception / warning
type of error message  $E$ 
try  $seq_1$ , evaluate  $seq_2$  on error
Functions with closure arguments / results
number of arguments of  $f$ 
select from  $v$  according to  $f$ 
apply  $f$  to all entries in  $v$ 
evaluate  $f(a_1, \dots, a_n)$ 
evaluate  $f(\dots f(f(a_1, a_2), a_3) \dots, a_n)$ 
calling function as closure
Sums & Products
sum  $X = a$  to  $X = b$ , initialized at  $x$ 
sum entries of vector  $v$ 
product of all vector entries
sum  $expr$  over divisors of  $n$ 
... assuming  $expr$  multiplicative
product  $a \leq X \leq b$ , initialized at  $x$ 
product over primes  $a \leq X \leq b$ 
Sorting
sort  $x$  by  $k$ -th component
min.  $m$  of  $x$  ( $m = x[i]$ ), max.
does  $y$  belong to  $x$ , sorted wrt.  $f$ 
 $\prod g^x \rightarrow$  factorization ( $\Rightarrow$  sorted, unique  $g$ )
Input/Output
print with/without  $\backslash n$ , TeX format
pretty print matrix
print fields with separator
formatted printing
write  $args$  to file
write  $x$  in binary format
read file into GP
... return as vector of lines
... return as vector of strings
read a string from keyboard
Files and file descriptors
File descriptors allow efficient small consecutive reads or writes
from or to a given file. The argument  $n$  below is always a descriptor,
attached to a file in r(ead), w(rite) or a(ppend) mode.
get descriptor  $n$  for file  $path$  in given  $mode$ 
... from shell  $cmd$  output (pipe)
close descriptor
commit pending write operations
read logical line from file
... raw line from file
write  $s \backslash n$  to file
... write  $s$  to file

```

```

forpart( $p = n, seq$ )
forperm( $S, p, seq$ )
forsubset( $n, p, seq$ )
forsubset( $[n, k], p, seq$ )
forqvec( $v, q, b, seq$ )
forsubgroup( $H = G$ )
until( $a, seq$ )
while( $a, seq$ )
break( $\{n\}$ )
next( $\{n\}$ )
return( $\{x\}$ )

error(), warning()
errname( $E$ )
iferr( $seq_1, E, seq_2$ )

Results
arity( $f$ )
select( $f, v$ )
apply( $f, v$ )
call( $f, a$ )
fold( $f, a$ )
self()

sum( $X = a, b, expr, \{x\}$ )
vecsum( $v$ )
vecprod( $v$ )
sumdiv( $n, X, expr$ )
sumdivmult( $n, X, expr$ )
prod( $X = a, b, expr, \{x\}$ )
prodeuler( $X = a, b, expr$ )

vecsort( $x, \{k\}, \{fl = 0\}$ )
vecmin( $x, \{\&i\}$ ), vecmax
vecsearch( $x, y, \{f\}$ )
matreduce( $m$ )

print, print1, printtex
printp
printsep( $sep, \dots$ ), printsep1
printf()

write, write1, writetex( $file, args$ )
writebin( $file, x$ )
read( $\{file\}$ )
readvec( $\{file\}$ )
readstr( $\{file\}$ )
input()

fileclose( $n$ )
fileflush( $n$ )
fileread( $n$ )
filereadstr( $n$ )
filewrite( $n, s$ )
filewrite1( $n, s$ )

```

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(PARI-GP version 2.15.3)

Timers

CPU time in ms and reset timer
CPU time in ms since gp startup
time in ms since UNIX Epoch
timeout command after s seconds

Interface with system

allocates a new stack of s bytes
alias old to new
install function from library
execute system command a
... and feed result to GP
... returning GP string
get \$VAR from environment
expand env. variable in string

```

gettime()
getabstime()
getwalltime()
alarm( $s, expr$ )

allocatemem( $\{s\}$ )
alias( $new, old$ )
install( $f, code, \{gpf\}, \{lib\}$ )
system( $a$ )
extern( $a$ )
externstr( $a$ )
getenv("VAR")
strexpand( $x$ )

```

Parallel evaluation

These functions evaluate their arguments in parallel (pthreads or MPI); args. must not access global variables (use **export** for this) and must be free of side effects. Enabled if threading engine is not *single* in gp header.

```

evaluate  $f$  on  $x[1], \dots, x[n]$ 
evaluate closures  $f[1], \dots, f[n]$ 
as select
as sum
as vector
eval  $f$  for  $i = a, \dots, b$ 
... for each element  $x$  in  $v$ 
... for  $p$  prime in  $[a, b]$ 
... for  $p = a \bmod q$ 
... multivariate
parforvec( $X = v, f, \{r\}, \{f_2\}, \{flag\}$ )
export  $x$  to parallel world
... all dynamic variables
frees exported value  $x$ 
... all exported values

```

```

parapply( $f, x$ )
pareval( $f$ )
parselect( $f, A, \{flag\}$ )
parsum( $i = a, b, expr$ )
parvector( $n, i, \{expr\}$ )
parfor( $i = a, \{b\}, f, \{r\}, \{f_2\}$ )
parforeach( $v, x, f, \{r\}, \{f_2\}$ )
parforprime( $p = a, \{b\}, f, \{r\}, \{f_2\}$ )
parforprimestep( $p = a, \{b\}, q, f, \{r\}, \{f_2\}$ )

```

Linear Algebra

dimensions of matrix x
multiply two matrices
... assuming result is diagonal
concatenation of x and y
extract components of x
transpose of vector or matrix x
adjoint of the matrix x
eigenvectors/values of matrix x
characteristic/minimal polynomial of x
trace/determinant of matrix x
permanent of matrix x
Frobenius form of x
QR decomposition
apply **matqr**'s transform to v

```

matsize( $x$ )
 $x * y$ 
matmultodiagonal( $x, y$ )
concat( $x, \{y\}$ )
vecextract( $x, y, \{z\}$ )
 $x \sim$ , mattranspose( $x$ )
matadjoint( $x$ )
mateigen( $x$ )
charpoly( $x$ ), minpoly( $x$ )
trace( $x$ ), matdet( $x$ )
matpermanent( $x$ )
matfrobenius( $x$ )
matqr( $x$ )
mathouseholder( $Q, v$ )

```

Constructors & Special Matrices

```

{ $g(x): x \in v$  s.t.  $f(x)$ }
{ $x: x \in v$  s.t.  $f(x)$ }
{ $g(x): x \in v$ }
row vec. of  $expr$  eval'ed at  $1 \leq i \leq n$ 
col. vec. of  $expr$  eval'ed at  $1 \leq i \leq n$ 
vector of small ints

```

```

[g(x) | x <- v, f(x)]
[x | x <- v, f(x)]
[g(x) | x <- v]
vector( $n, \{i\}, \{expr\}$ )
vectorv( $n, \{i\}, \{expr\}$ )
vectorsmall( $n, \{i\}, \{expr\}$ )

```

```

[ $c, c \cdot x, \dots, c \cdot x^n$ ]
[ $1, 2^x, \dots, n^x$ ]
matrix  $1 \leq i \leq m, 1 \leq j \leq n$ 
define matrix by blocks
diagonal matrix with diagonal  $x$ 
is  $x$  diagonal?
 $x \cdot \text{matdiagonal}(d)$ 
 $n \times n$  identity matrix
Hessenberg form of square matrix  $x$ 
 $n \times n$  Hilbert matrix  $H_{ij} = (i + j - 1)^{-1}$ 
 $n \times n$  Pascal triangle
companion matrix to polynomial  $x$ 
Sylvester matrix of  $x$  and  $y$ 

```

Gaussian elimination

```

kernel of matrix  $x$ 
intersection of column spaces of  $x$  and  $y$ 
solve  $MX = B$  ( $M$  invertible)
one sol of  $M * X = B$ 
basis for image of matrix  $x$ 
columns of  $x$  not in matimage
supplement columns of  $x$  to get basis
rows, cols to extract invertible matrix
rank of the matrix  $x$ 
solve  $MX = B \bmod D$ 
image mod  $D$ 
kernel mod  $D$ 
inverse mod  $D$ 
determinant mod  $D$ 

```

Lattices & Quadratic Forms

Quadratic forms

```

evaluate  ${}^t x Q y$ 
evaluate  ${}^t x Q x$ 
signature of quad form  ${}^t y * x * y$ 
decomp into squares of  ${}^t y * x * y$ 
eigenvalues/vectors for real symmetric  $x$ 

```

HNF and SNF

```

upper triangular Hermite Normal Form
HNF of  $x$  where  $d$  is a multiple of  $\det(x)$ 
multiple of  $\det(x)$ 
HNF of  $(x \mid \text{diagonal}(D))$ 
elementary divisors of  $x$ 
 $q$ -rank from elementary divisors
elementary divisors of  $\mathbf{Z}[a]/(f'(a))$ 
integer kernel of  $x$ 
 $\mathbf{Z}$ -module  $\leftrightarrow$   $\mathbf{Q}$ -vector space

```

Lattices

```

LLL-algorithm applied to columns of  $x$ 
... for Gram matrix of lattice
find up to  $m$  sols of qfnorm( $x, y) \leq b$ 
 $v, v[i] :=$  number of  $y$  s.t. qfnorm( $x, y) = i$ 
perfection rank of  $x$ 
find isomorphism between  $q$  and  $Q$ 
precompute for isomorphism test with  $q$ 
automorphism group of  $q$ 

```

Based on an earlier version by Joseph H. Silverman
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Send comments and corrections to (Karim.Belabas@math.u-bordeaux.fr)

convert **qfauto** for GAP/Magma **qfautoexport**($G, \{flag\}$)
orbits of V under $G \subset \text{GL}(V)$ **qforbits**(G, V)

Polynomials & Rational Functions

all defined polynomial variables **variables**()
get var. of highest priority (higher than v) **varhigher**($name, \{v\}$)
... of lowest priority (lower than v) **varlower**($name, \{v\}$)

Coefficients, variables and basic operators

degree of f **poldegree**(f)
coef. of degree n of f , leading coef. **polcoef**(f, n), **pollead**
main variable / all variables in f **variable**(f), **variables**(f)
replace x by y in f **subst**(f, x, y)
evaluate f replacing vars by their value **eval**(f)
replace polynomial expr. $T(x)$ by y in f **substpol**(f, T, y)
replace x_1, \dots, x_n by y_1, \dots, y_n in f **substvec**(f, x, y)

$f \in A[x]$; reciprocal polynomial $x^{\deg f} f\left(\frac{1}{x}\right)$ **polrecip**(f)
gcd of coefficients of f **content**(f)
derivative of f w.r.t. x **deriv**($f, \{x\}$)
... n -th derivative of f **derivn**($f, n, \{x\}$)
formal integral of f w.r.t. x **intformal**($f, \{x\}$)
formal sum of f w.r.t. x **sumformal**($f, \{x\}$)

Constructors & Special Polynomials

interpolation polynomial at $(x[1], y[1]), \dots, (x[n], y[n])$, evaluated at t , with error estimate e **polinterpolate**($x, \{y\}, \{t\}, \{&e\}$)
 $T_n/U_n, H_n$ **polchebyshev**(n), **polhermite**(n)
 $P_n, L_n^{(\alpha)}$ **pollegendre**(n), **pollaguerre**(n, a)
 n -th cyclotomic polynomial Φ_n **polcyclo**(n)
return n if $f = \Phi_n$, else 0 **poliscyclo**(f)
is f a product of cyclotomic polynomials? **poliscycloprod**(f)
Zagier's polynomial of index (n, m) **polzagier**(n, m)

Resultant, elimination

discriminant of polynomial f **poldisc**(f)
find factors of **poldisc**(f) **poldiscfactors**(f)
resultant $R = \text{Res}_v(f, g)$ **polresultant**($f, g, \{v\}$)
 $[u, v, R], xu + yv = \text{Res}_v(f, g)$ **polresultanttext**($x, y, \{v\}$)
solve Thue equation $f(x, y) = a$ **thue**($t, a, \{sol\}$)
initialize t for Thue equation solver **thueinit**(f)

Roots and Factorization (Complex/Real)

complex roots of f **polroots**(f)
bound complex roots of f **polrootsbound**(f)
number of real roots of f (in $[a, b]$) **polsturm**($f, \{[a, b]\}$)
real roots of f (in $[a, b]$) **polrootsreal**($f, \{[a, b]\}$)
complex embeddings of **t_POLMOD** z **convec**(z)

Roots and Factorization (Finite fields)

factor f mod p , roots **factormod**(f, p), **polrootsmod**
factor f over $\mathbf{F}_p[x]/(T)$, roots **factormod**($f, [T, p]$), **polrootsmod**
squarefree factorization of f in $\mathbf{F}_q[x]$ **factormodSQF**($f, \{D\}$)
distinct degree factorization of f in $\mathbf{F}_q[x]$ **factormodDDF**($f, \{D\}$)
factor n -th cyclotomic pol. Φ_n mod p **factormodcyclo**(n, p)

Roots and Factorization (p -adic fields)

factor f over \mathbf{Q}_p , roots **factorpadic**(f, p, r), **polrootspadic**
 p -adic root of f congruent to a mod p **padicappr**(f, a)
Newton polygon of f for prime p **newtonpoly**(f, p)
Hensel lift $A/\text{lc}(A) = \prod_i B[i]$ mod p^e **polhensellift**(A, B, p, e)
 $T = \prod (x - z_i) \mapsto \prod [x - \omega(z_i)] \in \mathbf{Z}_p[x]$ **polteichmuller**(T, p, e)
extensions of \mathbf{Q}_p of degree N **padicfields**(p, N)

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Roots and Factorization (Miscellaneous)

symmetric powers of roots of f up to n **polsym**(f, n)
Graeffe transform of $f, g(x^2) = f(x)f(-x)$ **polgraeffe**(f)
factor f over coefficient field **factor**(f)
cyclotomic factors of $f \in \mathbf{Q}[X]$ **polcyclofactors**(f)

Finite Fields

A finite field is encoded by any element (**t_FFELT**).
find irreducible $T \in \mathbf{F}_p[x]$, $\deg T = n$ **ffinit**($p, n, \{x\}$)
Create t in $\mathbf{F}_q \simeq \mathbf{F}_p[t]/(T)$ **t = ffgn**($T, 't$)
... indirectly, with implicit T **t = ffgn**($q, 't$); **T = t.mod**
map m from $\mathbf{F}_q \ni a$ to $\mathbf{F}_{q^k} \ni b$ **m = ffgend**(a, b)
build $K = \mathbf{F}_q[x]/(P)$ extending $\mathbf{F}_q \ni a$, **ffextend**(a, P)
evaluate map m on x **ffmap**(m, x)
inverse map of m **ffinvmap**(m)
compose maps $m \circ n$ **ffcompomap**(m, n)
 x as polmod over codomain of map m **ffmaprel**(m, x)
 F^n over $\mathbf{F}_q \ni a$ **fffrobenius**(a, n)
 $\#$ {monic irred. $T \in \mathbf{F}_q[x]$, $\deg T = n$ } **ffnbirred**(q, n)

Formal & p -adic Series

truncate power series or p -adic number **truncate**(x)
valuation of x at p **valuation**(x, p)
Dirichlet and Power Series
Taylor expansion around 0 of f w.r.t. x **taylor**(f, x)
Laurent series of closure F up to x^k **laurentseries**(f, k)
 $\sum a_k b_k t^k$ from $\sum a_k t^k$ and $\sum b_k t^k$ **serconvol**(a, b)
 $f = \sum a_k t^k$ from $\sum (a_k/k!) t^k$ **serlaplace**(f)
reverse power series F so $F(f(x)) = x$ **serreverse**(f)
remove terms of degree $< n$ in f **serchop**(f, n)
Dirichlet series multiplication / division **dirmul**, **dirdiv**(x, y)
Dirichlet Euler product (b terms) **direuler**($p = a, b, expr$)

Transcendental and p -adic Functions

real, imaginary part of x **real**(x), **imag**(x)
absolute value, argument of x **abs**(x), **arg**(x)
square/ n th root of x **sqrt**(x), **sqrtn**($x, n, \{&z\}$)
all n -th roots of 1 **rootsof1**(n)
FFT of $[f_0, \dots, f_{n-1}]$ **w = fftinit**(n), **fft/fftin**(w, f)
trig functions **sin**, **cos**, **tan**, **cotan**, **sinc**
inverse trig functions **asin**, **acos**, **atan**
hyperbolic functions **sinh**, **cosh**, **tanh**, **cotanh**
inverse hyperbolic functions **asinh**, **acosh**, **atanh**
 $\log(x)$, $\log(1+x)$, e^x , $e^x - 1$ **log**, **loglp**, **exp**, **expm1**
Euler Γ function, $\log \Gamma$, Γ'/Γ **gamma**, **lngamma**, **psi**
half-integer gamma function $\Gamma(n+1/2)$ **gammah**(n)
Riemann's zeta $\zeta(s) = \sum n^{-s}$ **zeta**(s)
 $\sum_{1 \leq n \leq N} n^s$ **dirpowerssum**(N, s)
Hurwitz's $\zeta(s, x) = \sum (n+x)^{-s}$ **zetahurwitz**(s, x)
Lerch $\Phi(z, s, x) = \sum z^n (n+x)^{-s}$ **lerchphi**(z, s, x)
Lerch $L(s, x, t) = \Phi(e^{2i\pi t}, s, x)$ **lerchzeta**(s, x, t)
multiple zeta value (MZV), $\zeta(s_1, \dots, s_k)$ **zetamult**($s, \{T\}$)
all MZVs for weight $\sum s_i = n$ **zetamultall**(n)
convert MZV id to $[s_1, \dots, s_k]$ **zetamultconvert**($f, \{flag\}$)
MZV dual sequence **zetamultdual**(s)
multiple polylog $Li_{s_1, \dots, s_k}(z_1, \dots, z_k)$ **polylogmult**(s, z)

incomplete Γ function ($y = \Gamma(s)$) **incgam**($s, x, \{y\}$)
complementary incomplete Γ **incgamc**(s, x)
 $\int_x^\infty e^{-t} dt/t$, $(2/\sqrt{\pi}) \int_x^\infty e^{-t^2} dt$ **eint1**, **erfc**
elliptic integral of 1st and 2nd kind **ellK**(k), **ellE**(k)
dilogarithm of x **dilog**(x)
 m -th polylogarithm of x **polylog**($m, x, \{flag\}$)
 U -confluent hypergeometric function **hyperu**(a, b, u)
Hypergeometric ${}_pF_q(A, B; z)$ **hypergeom**(A, B, z)
Bessel $J_n(x)$, $J_{n+1/2}(x)$ **besselj**(n, x), **besseljh**(n, x)
Bessel $I_\nu, K_\nu, H_\nu^1, H_\nu^2, Y_\nu$ **(bessel)i, k, h1, h2, y**
 k -th zero of $J_\nu(x)$ **besseljzero**($nu, \{k=1\}$)
 k -th zero of $Y_\nu(x)$ **besselyzero**($nu, \{k=1\}$)
Airy functions $A_i(x)$, $B_i(x)$ **airy**(x)
Lambert W : x s.t. $xe^x = y$ **lambertw**(y)
Teichmuller character of p -adic x **teichmuller**(x)

Iterations, Sums & Products

Numerical integration for meromorphic functions

Behaviour at endpoint for Double Exponential (DE) methods: either a scalar ($a \in \mathbf{C}$, regular) or $\pm\infty$ (decreasing at least as x^{-2}) or
 $(x-a)^{-\alpha}$ singularity **[a, a]**
exponential decrease $e^{-\alpha|x|}$ **[$\pm\infty, a$], $\alpha > 0$**
slow decrease $|x|^\alpha$ **$\dots \alpha < -1$**
oscillating as $\cos(kx)$ **$\alpha = k\mathbf{I}, k > 0$**
oscillating as $\sin(kx)$ **$\alpha = -k\mathbf{I}, k > 0$**

numerical integration **intnum**($x = a, b, f, \{T\}$)
weights T for **intnum** **intnuminit**($a, b, \{m\}$)
weights T incl. kernel K **intfuncinit**($t = a, b, K, \{m\}$)
integrate $(2i\pi)^{-1} f$ on circle $|z-a| = R$ **intcirc**($x = a, R, f, \{T\}$)
Other integration methods
 n -point Gauss-Legendre **intnumgauss**($x = a, b, f, \{n\}$)
weights for n -point Gauss-Legendre **intnumgaussinit**($\{n\}$)
quasi-periodic function, period $2H$ **intnumosc**($x = a, f, H$)
Romberg (low accuracy) **intnumromb**($x = a, b, f, \{flag\}$)

Numerical summation

sum of series $f(n)$, $n \geq a$ (low accuracy) **suminf**($n = a, expr$)
sum of alternating/positive series **sumalt**, **sumpos**
sum of series using Euler-Maclaurin **sumnum**($n = a, f, \{T\}$)
... Sidi summation **sumnumsidi**($n = a, f$)
 $\sum_{n \geq a} F(n)$, F rational function **sumnumrat**(F, a)
 $\dots \sum_{p \geq a} F(p^s)$ **sumeulerrat**($F, \{s=1\}, \{a=2\}$)
weights for **sumnum**, a as in DE **sumnuminit**($\{\infty, a\}$)
sum of series by Monien summation **sumnummonien**($n = a, f, \{T\}$)
weights for **sumnummonien** **sumnummonieninit**($\{\infty, a\}$)
sum of series using Abel-Plana **sumnumap**($n = a, f, \{T\}$)
weights for **sumnumap**, a as in DE **sumnumapinit**($\{\infty, a\}$)
sum of series using Lagrange **sumnumlagrange**($n = a, f, \{T\}$)
weights for **sumnumlagrange** **sumnumlagrangeinit**

Products

product $a \leq X \leq b$, initialized at x **prod**($X = a, b, expr, \{x\}$)
product over primes $a \leq X \leq b$ **prodeuler**($X = a, b, expr$)
infinite product $a \leq X \leq \infty$ **prodinf**($X = a, expr$)
 $\prod_{n \geq a} F(n)$, F rational function **prodnumrat**(F, a)
 $\prod_{p \geq a} F(p^s)$ **prodeulerrat**($F, \{s=1\}, \{a=2\}$)

Other numerical methods

| | |
|---|---|
| real root of f in $[a, b]$; bracketed root | <code>solve($X = a, b, f$)</code> |
| ...interval splitting, step s | <code>solvestep($X = a, b, s, f, \{flag = 0\}$)</code> |
| limit of $f(t)$, $t \rightarrow \infty$ | <code>limitnum($f, \{\alpha\}$)</code> |
| asymptotic expansion of f (rational) | <code>asypnum($f, \{\alpha\}$)</code> |
| ... $N + 1$ terms as floats | <code>asypnumraw($f, N, \{\alpha\}$)</code> |
| numerical derivation w.r.t x : $f'(a)$ | <code>derivnum($x = a, f$)</code> |
| evaluate continued fraction F at t | <code>contfraceval($F, t, \{L\}$)</code> |
| power series to cont. fraction (L terms) | <code>contfracinit($S, \{L\}$)</code> |
| Padé approximant (deg. denom. $\leq B$) | <code>bestapprPade($S, \{B\}$)</code> |

Elementary Arithmetic Functions

| | |
|--|---|
| vector of binary digits of $ x $ | <code>binary(x)</code> |
| bit number n of integer x | <code>bittest(x, n)</code> |
| Hamming weight of integer x | <code>hammingweight(x)</code> |
| digits of integer x in base B | <code>digits($x, \{B = 10\}$)</code> |
| sum of digits of integer x in base B | <code>sumdigits($x, \{B = 10\}$)</code> |
| integer from digits | <code>fromdigits($v, \{B = 10\}$)</code> |
| ceiling/floor/fractional part | <code>ceil, floor, frac</code> |
| round x to nearest integer | <code>round($x, \{\&e\}$)</code> |
| truncate x | <code>truncate($x, \{\&e\}$)</code> |
| gcd/LCM of x and y | <code>gcd(x, y), lcm(x, y)</code> |
| gcd of entries of a vector/matrix | <code>content(x)</code> |

Primes and Factorization

| | |
|--|---|
| extra prime table | <code>addprimes()</code> |
| add primes in v to prime table | <code>addprimes(v)</code> |
| remove primes from prime table | <code>removeprimes(v)</code> |
| Chebyshev $\pi(x)$, n -th prime p_n | <code>primepi(x), prime(n)</code> |
| vector of first n primes | <code>primes(n)</code> |
| smallest prime $\geq x$ | <code>nextprime(x)</code> |
| largest prime $\leq x$ | <code>precprime(x)</code> |
| factorization of x | <code>factor($x, \{lim\}$)</code> |
| ...selecting specific algorithms | <code>factorint($x, \{flag = 0\}$)</code> |
| $n = df^2$, d squarefree/fundamental | <code>core($n, \{fl\}$), coredisc</code> |
| certificate for (prime) N | <code>primecert(N)</code> |
| verifies a certificate c | <code>primecertisvalid(c)</code> |
| convert certificate to Magma/PRIMO | <code>primecertexport</code> |
| recover x from its factorization | <code>factorback($f, \{e\}$)</code> |
| $x \in \mathbf{Z}$, $ x \leq X$, $\gcd(N, P(x)) \geq N$ | <code>zncoppersmith($P, N, X, \{B\}$)</code> |
| divisors of N in residue class r mod s | <code>divisorslensstra(N, r, s)</code> |

Divisors and multiplicative functions

| | |
|--|---|
| number of prime divisors $\omega(n)$ / $\Omega(n)$ | <code>omega(n), bigomega</code> |
| divisors of n / number of divisors $\tau(n)$ | <code>divisors(n), numdiv</code> |
| sum of (k -th powers of) divisors of n | <code>sigma($n, \{k\}$)</code> |
| Möbius μ -function | <code>moebius(x)</code> |
| Ramanujan's τ -function | <code>ramanujantau(x)</code> |

Combinatorics

| | |
|---|---|
| factorial of x | <code>x!</code> or <code>factorial(x)</code> |
| binomial coefficient $\binom{x}{k}$ | <code>binomial($x, \{k\}$)</code> |
| Bernoulli number B_n as real/rational | <code>bernreal(n), bernfrac</code> |
| $[B_0, B_2, \dots B_{2k}]$ | <code>bernvec(k)</code> |
| Bernoulli polynomial $B_n(x)$ | <code>bernpol($n, \{x\}$)</code> |
| Euler numbers | <code>eulerfrac, eulerreal, eulervec</code> |
| Euler polynomial $E_n(x)$ | <code>eulerpol($n, \{x\}$)</code> |
| Eulerian polynomial $A_n(x)$ | <code>eulerianpol</code> |
| Fibonacci number F_n | <code>fibonacci(n)</code> |
| Harmonic number $H_{n,r} = 1^{-r} + \dots + n^{-r}$ | <code>harmonic(n, r)</code> |
| Stirling numbers $s(n, k)$ and $S(n, k)$ | <code>stirling($n, k, \{flag\}$)</code> |

Pari-GP reference card

(PARI-GP version 2.15.3)

| | |
|---|---|
| number of partitions of n | <code>numbpart(n)</code> |
| k -th permutation on n letters | <code>numtoperm(n, k)</code> |
| ...index k of permutation v | <code>permtounum(v)</code> |
| order of permutation p | <code>permorder(p)</code> |
| signature of permutation p | <code>permsign(p)</code> |
| cyclic decomposition of permutation p | <code>permcycles(p)</code> |

Multiplicative groups $(\mathbf{Z}/N\mathbf{Z})^*$, \mathbf{F}_q^*

| | |
|---|--|
| Euler ϕ -function | <code>eulerphi(x)</code> |
| multiplicative order of x (divides ϕ) | <code>znorder($x, \{o\}$), fforder</code> |
| primitive root mod q / x .mod | <code>znprimroot(q), fprimroot(x)</code> |
| structure of $(\mathbf{Z}/n\mathbf{Z})^*$ | <code>znstar(n)</code> |
| discrete logarithm of x in base g | <code>znlog($x, g, \{o\}$), fflag</code> |
| Kronecker-Legendre symbol $(\frac{x}{y})$ | <code>kronecker(x, y)</code> |
| quadratic Hilbert symbol (at p) | <code>hilbert($x, y, \{p\}$)</code> |

Euclidean algorithm, continued fractions

| | |
|--|---|
| CRT: solve $z \equiv x$ and $z \equiv y$ | <code>chinese(x, y)</code> |
| minimal u, v so $xu + yv = \gcd(x, y)$ | <code>gcdext(x, y)</code> |
| half-gcd algorithm | <code>halfgcd(x, y)</code> |
| continued fraction of x | <code>confrac($x, \{b\}, \{lmax\}$)</code> |
| last convergent of continued fraction x | <code>confracpnqn(x)</code> |
| rational approximation to x (den. $\leq B$) | <code>bestappr($x, \{B\}$)</code> |
| recognize $x \in \mathbf{C}$ as polmod mod $T \in \mathbf{Z}[X]$ | <code>bestapprnf(x, T)</code> |

Miscellaneous

| | |
|---|---|
| integer square / n -th root of x | <code>sqrtint(x), sqrtsint(x, n)</code> |
| largest integer e s.t. $b^e \leq b$, $e = \lfloor \log_b(x) \rfloor$ | <code>logint($x, b, \{\&z\}$)</code> |

Characters

Let $cyc = [d_1, \dots, d_k]$ represent an abelian group $G = \oplus (\mathbf{Z}/d_j\mathbf{Z}) \cdot g_j$ or any structure G affording a .cyc method; e.g. `znstar($q, 1$)` for Dirichlet characters. A character χ is coded by $[c_1, \dots, c_k]$ such that $\chi(g_j) = e(n_j/d_j)$.
 $\chi \cdot \psi$; χ^{-1} ; $\chi \cdot \psi^{-1}$; χ^k `charmul, charconj, chardiv, charpow`
order of χ `charorder(cyc, χ)`
kernel of χ `charker(cyc, χ)`
 $\chi(x)$, G a GP group structure `chareval($G, \chi, x, \{z\}$)`
Galois orbits of characters `chargalois(G)`

Dirichlet Characters

| | |
|---|---|
| initialize $G = (\mathbf{Z}/q\mathbf{Z})^*$ | <code>G = znstar($q, 1$)</code> |
| convert datum D to $[G, \chi]$ | <code>znchar(D)</code> |
| is χ odd? | <code>zncharisodd(G, χ)</code> |
| real $\chi \rightarrow$ Kronecker symbol (D/\cdot) | <code>znchartokronecker(G, χ)</code> |
| conductor of χ | <code>zncharconductor(G, χ)</code> |
| $[G_0, \chi_0]$ primitive attached to χ | <code>znchartoprimitive(G, χ)</code> |
| induce $\chi \in \hat{G}$ to $\mathbf{Z}/N\mathbf{Z}$ | <code>zncharinduce(G, χ, N)</code> |
| χp | <code>znchardecompose(G, χ, p)</code> |
| $\prod_p (Q, N) \chi p$ | <code>znchardecompose(G, χ, Q)</code> |
| complex Gauss sum $G_a(\chi)$ | <code>znchargauss(G, χ)</code> |

Conrey labelling

| | |
|---|--|
| Conrey label $m \in (\mathbf{Z}/q\mathbf{Z})^* \rightarrow$ character | <code>znconreychar(G, m)</code> |
| character \rightarrow Conrey label | <code>znconreyexp(G, χ)</code> |
| log on Conrey generators | <code>znconreylog(G, m)</code> |
| conductor of χ (χ_0 primitive) | <code>znconreyconductor($G, \chi, \{\chi_0\}$)</code> |

True-False Tests

| | |
|--|--|
| is x the disc. of a quadratic field? | <code>isfundamental(x)</code> |
| is x a prime? | <code>isprime(x)</code> |
| is x a strong pseudo-prime? | <code>ispseudoprime(x)</code> |
| is x square-free? | <code>issquarefree(x)</code> |
| is x a square? | <code>issquare($x, \{\&n\}$)</code> |
| is x a perfect power? | <code>ispower($x, \{k\}, \{\&n\}$)</code> |
| is x a perfect power of a prime? ($x = p^n$) | <code>isprimepower($x, \&n$)</code> |
| ... of a pseudoprime? | <code>ispseudoprimepower($x, \&n$)</code> |
| is x powerful? | <code>ispowerful(x)</code> |
| is x a totient? ($x = \varphi(n)$) | <code>istotient($x, \{\&n\}$)</code> |
| is x a polygonal number? ($x = P(s, n)$) | <code>ispolygonal($x, s, \{\&n\}$)</code> |
| is pol irreducible? | <code>polisirreducible(pol)</code> |

Graphic Functions

| | |
|---|---|
| crude graph of $expr$ between a and b | <code>plot($X = a, b, expr$)</code> |
| High-resolution plot (immediate plot) | <code>plotth($X = a, b, expr, \{flag\}, \{n\}$)</code> |
| plot $expr$ between a and b | <code>plotth($X = a, b, expr, \{flag\}, \{n\}$)</code> |
| plot points given by lists lx, ly | <code>plotthraw($lx, ly, \{flag\}$)</code> |
| terminal dimensions | <code>plotsizes()</code> |
| Rectwindow functions | |
| init window w , with size x, y | <code>plotinit(w, x, y)</code> |
| erase window w | <code>plotkill(w)</code> |
| copy w to w_2 with offset (dx, dy) | <code>plotcopy(w, w_2, dx, dy)</code> |
| slice contents of w | <code>plotclip(w)</code> |
| scale coordinates in w | <code>plotscale(w, x_1, x_2, y_1, y_2)</code> |
| plotth in w | <code>plotrecth($w, X = a, b, expr, \{flag\}, \{n\}$)</code> |
| plotthraw in w | <code>plotrecthraw($w, data, \{flag\}$)</code> |
| draw window w_1 at $(x_1, y_1), \dots$ | <code>plotdraw($[[w_1, x_1, y_1], \dots]$)</code> |

Low-level Rectwindow Functions

| | |
|---|--|
| set current drawing color in w to c | <code>plotcolor(w, c)</code> |
| current position of cursor in w | <code>plotcursor(w)</code> |
| write s at cursor's position | <code>plotstring(w, s)</code> |
| move cursor to (x, y) | <code>plotmove(w, x, y)</code> |
| move cursor to $(x + dx, y + dy)$ | <code>plotrmove(w, dx, dy)</code> |
| draw a box to (x_2, y_2) | <code>plotbox(w, x_2, y_2)</code> |
| draw a box to $(x + dx, y + dy)$ | <code>plotrbox(w, dx, dy)</code> |
| draw polygon | <code>plotlines($w, lx, ly, \{flag\}$)</code> |
| draw points | <code>plotpoints(w, lx, ly)</code> |
| draw line to $(x + dx, y + dy)$ | <code>plotrline(w, dx, dy)</code> |
| draw point $(x + dx, y + dy)$ | <code>plotrpoint(w, dx, dy)</code> |

Convert to Postscript or Scalable Vector Graphics

| | |
|---|--|
| The format f is either "ps" or "svg". | |
| as plotth | <code>plotthexport($f, X = a, b, expr, \{flag\}, \{n\}$)</code> |
| as plotthraw | <code>plotthrawexport($f, lx, ly, \{flag\}$)</code> |
| as plotdraw | <code>plotexport($f, [[w_1, x_1, y_1], \dots]$)</code> |